19. The Wave Nature of Electrons

- **interpret electron diffraction patterns as evidence for the wave-like nature of matter**
- **distinguish between the diffraction patterns produced by photons and electrons**
- **calculate the de Broglie wavelength of matter: $\lambda = \frac{h}{p}$**
- **compare the momentum of photons and of matter of the same wavelength including calculations using: $p = \frac{h}{\lambda}$**
- **investigate and describe theoretically and practically the effects of varying the width of a gap or diameter of an obstacle on the diffraction pattern produced by light and apply this to limitations of imaging using light**
- **explain how diffraction from a single slit experiment can be used to illustrate Heisenberg's uncertainty principle**

Units
Multiplying prefixes may be used with any unit symbols to indicate decimal multiples or fractions.

10^{24}	10^{21}	10^{18}	10^{15}	10^{12}	10^{9}	10^{6}	10^{3}	10^{0}
Yotta	Zetta	Exa	Peta	Tera	Giga	Mega	Kilo	
Y	Z	E	P	T	G	M	k	

10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}	10^{-18}	10^{-21}	10^{-24}
milli	micro	nano	pico	femto	atto	zepto	yocto
m	μ	n	p	f	a	z	y

If electrons can behave as waves inside an atom, they might be able to exhibit other wave properties, like interference. This was shown in electron scattering experiments - definite patterns of reinforcement (antinodes) and reduction (nodes) were found in scattering. The gaps that the electrons were passing through in scattering were very small, in the order of the diameter of an atom. We can control the wavelength of the light by controlling the momentum of the electron.

The wave-like nature of matter

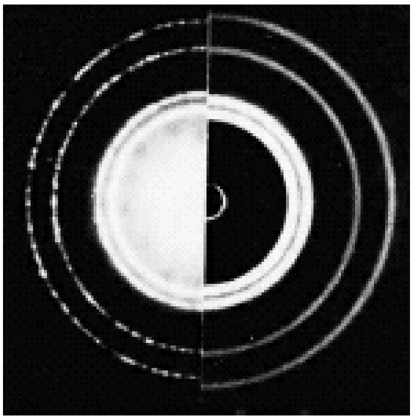
In 1923 Louis de Bröglie speculated that if light waves could behave like particles, then particles of matter should behave like waves. Experiments with electrons clearly show that they can diffract and interfere with each other. Protons and neutrons also have been shown to exhibit wave-like behaviour. De Bröglie suggested that the wavelength of a particle of matter could be found by using the same relationship that applies to photons, i.e. $p = \frac{h}{\lambda}$

de Bröglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

The wavelength is inversely proportional to the momentum. With any mass that is not sub-atomic, the product of 'mv' is so large that λ is always of the order 10^{-33}m. This is too small for us to see.

Electron Diffraction Patterns

One experiment that is used to show the wave nature of electrons is the electron diffraction pattern. Electrons are sent through a piece of metal as they pass through they leave a pattern on a screen. Only half of the pattern is projected onto the screen, then x-rays are sent through the metal and the pattern from the x-rays is projected onto the other half of the screen. If the two patterns align then the x-rays must have the same wavelength as the electrons.
The pattern is formed from diffraction as the wave fronts travel between the atoms. Then the wavefronts interfere with each other to form bright and dark rings.



electrons X-rays

Quantum Physics

Max Planck proposed that energy travels in discrete packets called quanta. Prior to Planck's work with black body radiation, energy was thought to be continuous, but this theory left many phenomena unexplained. While working out the mathematics for the radiation phenomena he had observed, Planck realised that quantised energy could explain the behaviour of light.

In 1900 Max Planck began to study the range of electromagnetic radiation that emanates from a very hot body (black body radiation). When a body is heated, it first glows red; with further heating it turns to white and eventually blue (ie. the wavelength of light emitted becomes shorter and it frequency becomes higher with increasing temperature).

Experiments showed that the amount of energy that is emitted falls off to zero at high and low wavelengths. As the temperature of the body changes, the hump in the energy distribution curve shift.

Planck attempted to use classical theory to explain the strange behaviour of this distribution curve. Classical theory predicted that the amount of energy emitted by a hot body should increase continuously as the frequency increases. Planck could not explain the observed energy distribution using classical theory, so he developed a new concept. This was that energy is radiated in discrete packets or bundles called **quanta** (singular: a quantum), rather than in continuous amounts as Wave Theory implies. The energy-radiated (*E*) is proportional to the frequency (*f*) of the radiation. The two quantities were connected by a proportionality constant (*h*).

$$E = \frac{hc}{\lambda} \qquad c = \text{speed of light.}$$

This is also written as **E = hf** where **f** is the frequency of the light.

Photons

The modern theory of light is a merging of the wave and the particle models. Light is imagined to travel in a stream of packets or *quanta* of energy. The energy carried by each quantum is proportional to the frequency of light and can be found from Planck's equation: E = hf. Greater intensity of light has more photons - each photon still has the same energy.

Photons are neither particles nor waves. They have a set of properties that have some similarities to particles when travelling through a vacuum and when in a gravity field, and some similarities to waves when refracting and interfering.

Wave-Particle duality of light

The paradoxical view of the nature of light was built into Einstein's original formulation of the photon model. The energy of the light was supposed to be carried in the form of a particle or photon, the energy of the photon was defined in terms of the frequency of the radiation, and frequency is an essentially wavelike property! One possible way of dealing with this paradox is to treat light as a wave some of the time and as a particle at other times.

The behaviour of waves varies across the electromagnetic spectrum. Low frequency photons such as radio-waves and microwaves exhibit distinctly wave-like behaviours such as diffraction and interference, but have no particle-like properties. Around the middle of the spectrum in the visible light region, photons have both wave and particle properties. They interfere and diffract like waves, and also interact with electrons in the photoelectric effect as particles do. At the high frequency end of the spectrum, X-ray and gamma ray photons behave much more like particles than waves.

The momentum of Photons

Maxwell suggested that photons do have momentum given by

$$p = \frac{E}{c}$$

where c is the speed of light and E is the energy of the photon.

As the energy of the photon is related to its frequency by Planck's equation, and, since $c = f \times \lambda$ for waves, the momentum equation can be written as

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$

De Bröglie argued that the universe should be symmetrical and that the equation $p = \frac{h}{\lambda}$ should apply to particles as well as waves.

A particle's momentum is given by $p = mv$ and so

$$p = mv = \frac{h}{\lambda} \qquad \therefore \lambda = \frac{h}{mv}$$

The wavelength calculated is called the De Bröglie wavelength for the particle

When sound and light waves pass through narrow slits, they show diffraction effects only when the slit is about 1 - 50 λ. Thus, a particle could be expected to show diffraction only if it is passing through a gap 1 - 50 times its De Bröglie wavelength. For most large particles this is impractical as the physical size of the particle is far too large. Only in the case of small particles such as electrons, is the De Bröglie wavelength enough for diffraction and interference effects to occur.

Properties of photons with similarities to waves and or particles

Speed depends on the medium	W	
Two light beams can pass through each other without interacting.	W	
When light passes from one medium to another, refraction occurs.	W	
Light diffracts when travelling through narrow openings	W	
Different colours correspond to different energies of light.	W	P
Reflection from a shiny surface: angles of incidence = angle of reflection.	W	P
Light exerts a pressure		P
Travels through a vacuum as well as transparent media.		P
Beams of light are bent (a little) by gravity.		P
Certain colours (energies) of light eject electrons from metal surfaces.		P

Heisenberg uncertainty principle

Quantum physics (not classical physics), assumes that a matter wave, like a light wave, is a probability wave. Therefore the probability (per unit time) of detecting a particle in a small volume centred on a given point in a matter wave is proportional to the value of the probability density squared at that point. (Erwin Schrödinger 1926).

In 1927 Werner Heisenberg proposed that measured values cannot be given to position (x) and momentum (p) of a particle simultaneously with unlimited precision.

This uncertainty is not related to the measurement techniques, or limitations on measurement devices, but it is an outcome of both wave-particle duality and the interactions between the object being observed and the effect of the observation on that object.

For the normal 'nonquantum' world Δx and Δp_x are so small they are considered insignificant, but at the atomic scale, this level of uncertainty is significant.

To measure the precise location of a free particle (e.g. electron), it needs to be hit with another particle (e.g. photon). This will cause the electron to move (or move differently) as energy is transferred from the photon. Therefore the act of measuring causes a change in the value of what is being measured.

If Δx is position uncertainty, and Δp momentum uncertainty then $\Delta x \times \Delta p_x \geq \frac{h}{4\pi}$. (Where h is Planck's constant 6.63×10^{-34} J s). If one uncertainty is small (→ 0) then the other is large (→ ∞) to maintain $\geq \frac{h}{4\pi}$.

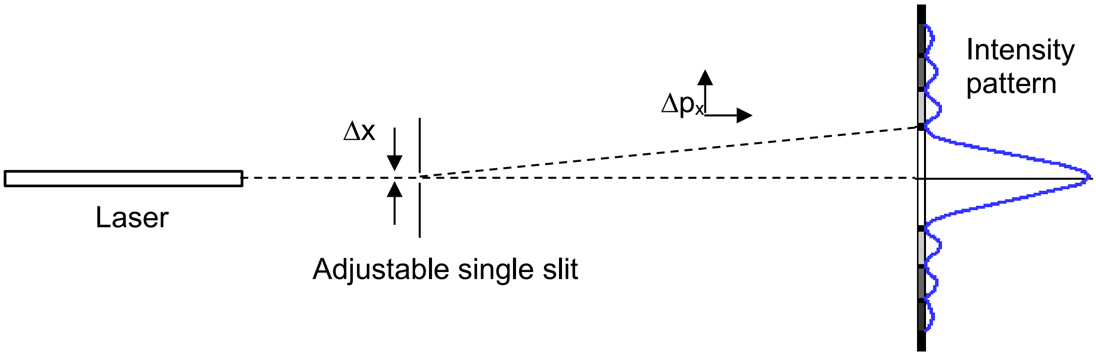
To find the minimum uncertainty allowed, use $\Delta x \times \Delta p_x = \frac{h}{4\pi}$.

As Δx decreases Δp_x has to increase.

Single slit diffraction.

In 1909, G I Taylor, carried out the single slit experiment using light so feeble that only one photon passed randomly through the slit at a time. Interference fringes built up on the screen (over 3 months), even though the photons could not have been interacting with each other.

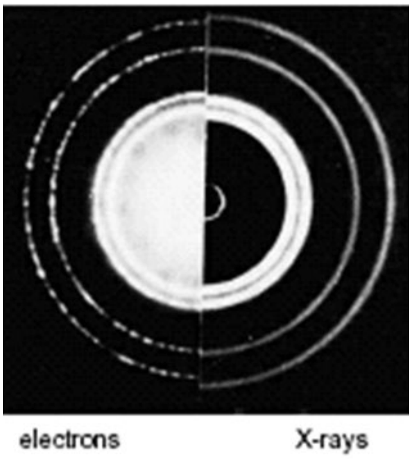
As the photon passes through the slit, Δx is the slit width, its position is known with some uncertainty. Heisenberg says that this introduces some uncertainty Δp_x, so beam spreads out, hence the interference pattern. If Δx is smaller, Δp_x must be greater so beam spreads out more.



Davisson and Germer (1928) demonstrated the wave nature of electrons, so single slit diffraction can also be observed with particles, e.g. electrons, protons, neutrons etc.

Solving problems with electrons and X – rays.

If you have a question where electrons and X – rays create the same diffraction pattern like below



that means that they have the same wavelength as diffraction pattern depends on the wavelength and so they have same momentum. Depends on what you know you should use the next formulas:

Electrons

$$p = mv = \sqrt{2mE_k}$$

$$E_k = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

X-rays photons

$$E = \frac{hc}{\lambda} = pc$$

And equalize either $p_{el} = p_{photon}\ or\ \lambda_{el} = \lambda_{photon}$