Universal law of gravity worksheet solutions.

For the questions in this worksheet take:
g (on the Earth's surface] = 9.8 m s^{-2}
G(the Universal Constant of Gravitation) = 6.67 x 10^{-11} N m^2 kg^{-2}
M_E (the mass of the Earth) = 6.0 x 10^{24} kg
M_M (the mass of the Moon) = 7.34 x 10^{22} kg
R_M (the radius of the Moon's orbit around the Earth) = 3.84 x 10^8 m
R_E (the radius of the Earth) = 6.38 x 10^6 m
r_M (the radius of the Moon) = 1.74 x 10^6 m 7
M_S (the mass of the Sun) = 2.0 x 10^{30}. kg
R_{SE} (the radius of the Earth's orbit around the Sun) = 1.5 x 10^{11} m

1. What is the gravitational force of attraction between:
a two oranges of mass 0.12 kg placed 0.20 m apart on a table?

$$F_g = \frac{Gm_1m_2}{r^2}$$
$$= \frac{(6.67\times10^{-11})(0.12\,\text{kg})(0.12\,\text{kg})}{(0.20\,\text{m})^2}$$
$$= 2.4\times10^{-11}\,\text{N}$$

b the Earth and the Sun?

$$F_g = \frac{Gm_1m_2}{r^2}$$
$$= \frac{(6.67\times10^{-11})(6.0\times10^{24}\,\text{kg})(2.0\times10^{30}\,\text{kg})}{(1.5\times10^{11}\,\text{m})^2}$$
$$= 3.6\times10^{22}\,\text{N}$$

2. How far apart would you theoretically need to place two masses each of 4.0 x 10^5 kg, in order for the force between them to be 0.10 N?

$$F_g = \frac{Gm_1m_2}{r^2}$$
$$0.10\,\text{N} = \frac{(6.67\times10^{-11})(4.0\times10^5\,\text{kg})^2}{r^2}$$
$$r = \sqrt{\frac{(6.67\times10^{-11})(4.0\times10^5\,\text{kg})^2}{0.10\,\text{N}}}$$
$$= 10.3\,\text{m}$$

3. The magnitude of the gravitational force of attraction between two objects is F when they are placed a distance r apart. What is the magnitude of the gravitational force of attraction between the two masses if:
a the mass of one object is halved?

$$\frac{Gm_1(m_2/2)}{r^2} = \frac{1}{2}\frac{Gm_1m_2}{r^2} = \frac{F}{2}$$

b the mass of both objects is halved?

$$\frac{G(m_1/2)(m_2/2)}{r^2} = \frac{1}{4}\frac{Gm_1m_2}{r^2} = \frac{F}{4}$$

c the distance between the centres of the two masses is halved?

$$\frac{Gm_1m_2}{(r/2)^2} = 4\frac{Gm_1m_2}{r^2} = 4F$$

d the distance between the two masses is increased by a factor of three?

$$\frac{Gm_1m_2}{(3r)^2} = \frac{1}{9}\frac{Gm_1m_2}{r^2} = \frac{F}{9}$$

4. What is the gravitational force of attraction between the Earth and the Moon?

$$F = \frac{Gm_e m_m}{(r)^2}$$

$$= \frac{(6.67\times10^{-11})(6.0\times10^{24}\ \text{kg})(7.34\times10^{22}\ \text{kg})}{(3.84\times10^{8}\ \text{m})^2}$$

$$= 2.0\times10^{20}\ \text{N}$$

5. a What is the weight of a 1.0 kg mass on the surface of the Earth?

9.8 N

b How far from the centre of the Earth is the mass when its weight is 5.0 N?

$$F_g = \frac{Gm_e m_m}{r^2}$$

$$= \frac{(6.67\times10^{-11})(6.0\times10^{24}\ \text{kg})(1.0\,\text{kg})}{r^2}$$

$$= 5.0\,\text{N}$$

$$r = \sqrt{\frac{(6.67\times10^{-11})(6.0\times10^{24}\ \text{kg})(1.0\,\text{kg})}{(5.0\,\text{N})}}$$

$$= 9.0\times10^{6}\ \text{m}$$

6. What is the gravitational field strength at a point whose distance from the Earth's surface is equal to the radius of the Earth?

$$F_g = \frac{Gm_e m_m}{(2r_e)^2} = \frac{1}{4}\frac{Gm_e m_m}{r_e^2} = \frac{10\,\text{N}\,\text{kg}^{-1}}{4}$$

$$= 2.5\,\text{N}\,\text{kg}^{-1}$$

7. What is the gravitational field strength on the surface of the Moon due to the Moon?

$$F_g = \frac{Gm_m m}{r_m^2} = mg_m$$

$$g_m = \frac{Gm_m}{r_m^2}$$

$$= \frac{(6.67\times10^{-11})(7.34\times10^{22}\ \text{kg})}{(1.74\times10^{6}\ \text{m})^2}$$

$$= 1.6\ \text{N kg}^{-1}$$

8. At what distance from the Earth would a spacecraft experience zero net gravitational force due to the opposing pulls of the Earth and the Moon?

Distance from Earth to Moon is r_{EM}

Let distance from Earth to Spacecraft be r_{ES}, therefore distance from Moon to Spacecraft r_{MS}, is $r_{EM} - r_{ES}$

When spacecraft experiences zero net force,

$$F_{ES} = F_{MS}$$

$$\frac{Gm_{Earth}\, m_{spacecraft}}{r_{ES}^2} = \frac{Gm_{Moon}\, m_{spacecraft}}{r_{MS}^2}$$

$$\frac{Gm_{Earth}\, m_{spacecraft}}{r_{ES}^2} = \frac{Gm_{Moon}\, m_{spacecraft}}{(r_{EM} - r_{ES})^2}$$

$$\frac{m_{Earth}}{r_{ES}^2} = \frac{m_{Moon}}{(r_{EM} - r_{ES})^2}$$

$$\frac{r_{ES}^2}{(r_{ES} - r_{ES})^2} = \frac{m_{Earth}}{m_{Moon}} = \frac{6.0\times10^{24}}{7.34\times10^{22}}$$

$$\frac{r_{ES}}{(r_{ES} - r_{ES})} = \sqrt{81.7} = 9.04$$

$$r_{ES} = 9.04(r_{EM} - r_{ES})$$

$$10.04 r_{ES} = 9.04 r_{EM}$$

$$r_{ES} = \frac{9.04 r_{EM}}{10.04} = \frac{9.04\times3.84\times10^{8}}{10.04}\ \text{m}$$

$$r_{ES} = 3.5\times10^{8}\ \text{m}$$