

1. You should be able to manipulate Lorentz's factor formula from

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \text{ to } \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

2. When solving problems with time dilation remember that proper time is the **shortest** time and that time doesn't slow down for moving object, it is ticking faster for stationary observer. Don't be confused with the words time running slower or faster. When it is said that time running slower it means that measured time interval will be smaller and it will be the proper time. When it is said that time running faster it means that measured time interval will be greater and it will be the dilated time.

$$t = t_0\gamma$$

3. Before solving the problem answer few questions:

1) Who is moving, who is stationary?

2) Who measure time for himself, who measure time for another?

Proper time will be the time measured by the moving object for itself, all other times will be dilated times.

4. To successfully solve the problems with length contraction answer simple question – do you measure length of something that is stationary relative to you or you measure length of the object which is moving relative to you. Also, remember that proper length is the largest length and that contraction appear only in the direction of motion, all other dimensions appear unchanged.

Let's consider the next example:

Tests of relativistic time dilation have been made by observing the decay of short-lived particles. A muon, travelling from the edge of the atmosphere to the surface of Earth, is an example of such a particle.

To model this in the laboratory, another elementary particle with a shorter half-life is produced in a particle accelerator. It is travelling at $0.99875c$ ($\gamma = 20$). Scientists observe that this particle travels $9.14 \times 10^{-5}\text{m}$ in a straight line from the point where it is made to the point where it decays into other particles. It is not accelerating.

a. Calculate the lifetime of the particle in the scientists' frame of reference.

As scientist is stationary relative to the accelerator time is just distance over speed.

$$t = \frac{d}{v} = \frac{9.14 \times 10^{-5}}{0.99875 \times 3 \times 10^8} = 3.05 \times 10^{-13}\text{s}$$

And this always will be the case $t = \frac{d}{v}$ when calculating the time measured by the stationary observer over the distance measured by the stationary observer and speed relative to the stationary observer.

b. (2 marks) 38%

Calculate the distance that the particle travels in the laboratory, as measured in the particle's frame of reference.

$$L = \frac{L_0}{\gamma} = \frac{9.14 \times 10^{-5}}{20} = 4.6 \times 10^{-6} \text{ m}$$

As particle is moving relative to the laboratory it will measure contracted length.

c. Explain why the scientists would observe more particles at the end of the laboratory measuring range than classical physics would expect.

This can be answered using either time dilation or length contraction (but **never** use both of them at the same time!).

Using length contraction:

Due to the particle's velocity distance to the detector in particle's frame of reference is reduced. Therefore more particles will be able to reach detector before they decay.

Using time dilation:

Due to the particle's velocity, its half-life as measured in the scientists' frame of reference is increased. Therefore, fewer particles will decay before reaching the detector and more particles will be detected.