Rotational equilibrium solutions

Question 1

torque, τ = 30 N m
distance, $r_\perp$ = 0.30 m
So force, $\mathbf{F} = \tau / r_\perp$
= 30 Nm/0.30 m
= 100 N

Question 2

Torque = $r_\perp \mathbf{F}$, so $r_\perp$ = 30/30 = 1 m

Question 3

a. To satisfy equilibrium, both the sum of the forces acting on the seesaw and the sum of the torques must equal zero.

Sum of forces up = sum of forces down
$R = 800 + 600$
R = 1400 N upwards.

b. Taking torques about the fulcrum at the centre:
Sum of clockwise torques = sum of anticlockwise torques
$600 \times 2 = 800 \times d$
d = 1.5 m.

Question 4

If the structure is stable, both the sum of forces and the sum of torques must equal zero.

From net force = zero:
Sum of forces up = sum of forces down
$R_1 + R_2 = 40 \times 9.8 + 60 \times 9.8$ N = 980 N

Taking torques about trestle 1:
Sum of clockwise torques = sum of anticlockwise torques

$40 \times 9.8 \times \frac{1}{4}L + 60 \times 9.8 \times \frac{1}{2}L = R_2 \times L$

Cancel L gives:
$98 + 294 = R_2$
R_2 = 392 N

Substituting into the first equation gives

$R_1 + 392$ = 980 N
R_1 = 588 N

Rotational equilibrium solutions

Question 5 2001 Question 1, 83%

Torque = F × r

where F = 600 N and r = 0.4 m

∴ torque = 600 × 0.4 ∴ τ = 240 N m

∴ τ = 240 N m (ANS)

Question 6 2003 Question 6, 51%

Using torques about support 1.

$(50 \times 10^3 \times 10 \times 4.5) + (300 \times 10^3 \times 10 \times 6.75) = F_2 \times 13.5$

∴ $F_2 = 1.67 \times 10^6$

As the system is in equilibrium.

ΣF = 0,

∴ $(50 \times 10^3 \times 10) + (300 \times 10^3 \times 10) = F_1 + F_2$

∴ $F_1 = 1.73 \times 10^6$ N and $F_2 = 1.67 \times 10^6$ N

Question 7 2002 Question 3, 53%

The standard way to calculate these is to consider:

Σforces up = Σforces down.

Σclockwise torque about any point = Σanti-clockwise torque.

Taking torques about the right-hand bridge support resulted in the torque equation:

$N_1 \times 30 = 20 \times 10^3 \times 10 \times 15 + 6 \times 10^3 \times 10 \times 10$

∴ $N_1 = 1.2 \times 10^5$ N (ANS)

Similarly, taking torques about the left-hand bridge support resulted in

$N_2 = 1.4 \times 10^5$ N. (ANS)

Alternately:

N_1 and N_2 are the forces that support the bridge AND the truck.

To support the bridge, each force needs to support 10 tonnes each.

To support the truck, N_2 must support 4 tonnes while N_1 must support only 2 tonnes, since the distance from N_1 is twice that of N_2.

So N_1 supports 12 tonnes, while N_2 supports 14 tonnes.

Using $g = 10\ m\ s^{-2}$ to convert each of these to Newton gives:

$N_1 = 120$ kN and $N_2 = 140$ kN (ANS)

Question 8 2001 Question 7, 57%

Since the prize is not accelerating, then the net force on the prize must be zero.

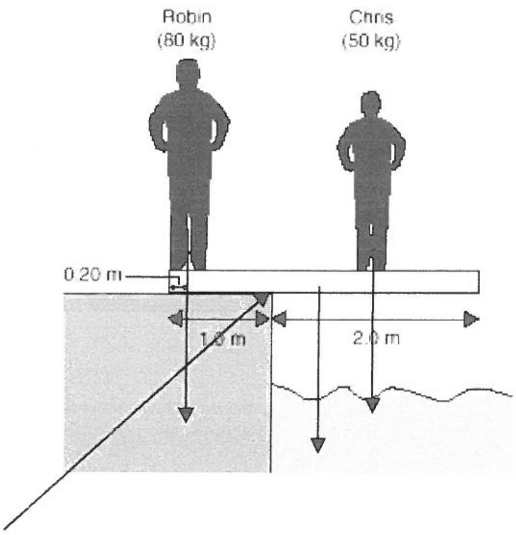
$\therefore$ 0 N (ANS)

Question 9 2001 Question 8, 57%

To balance the vertical forces, $T = mg$ $\therefore T = 1.0 \times 10$

$\therefore T = 10$ N (ANS)

Question 10 2001 Question 9, 38%



Rotational equilibrium solutions

Take the torques about this point.

Use this point because it is the actual point that the plank will rotate around. It also eliminates one unknown force, i.e. the thrust from the ground.

There will be three torques acting.

The torque due to Robin, Chris and the beam.

The anti-clockwise torque will be:

$80 \times 10 \times 0.8 = 640$

The clockwise torques will be:

$40 \times 10 \times 0.5 + 50 \times 10 \times d$

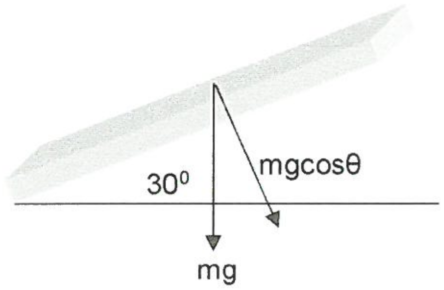
$200 + 500 \times d$

This will give $640 - 200 = 500 \times d$

$$\therefore d = \frac{440}{500}$$

$\therefore$ d = 0.88 m (ANS)

Question 11 2000 Question 5, 30%



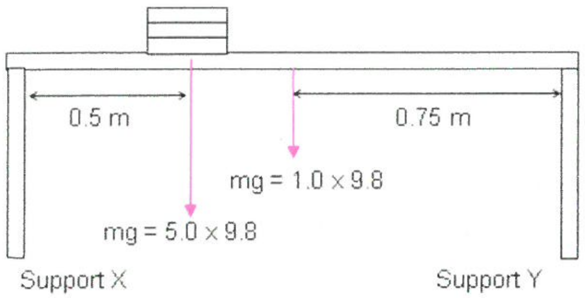
To do this question you have to multiply the radius by the force that act at right angles to the beam.

$\tau = Fr\cos\theta$

$\tau = 107 \times 9.8 \times 35 \times \cos30$

$\boldsymbol{\tau = 3.2 \times 10^7}$ **N m (ANS)**

Question 12 1999 Question 3, 70%

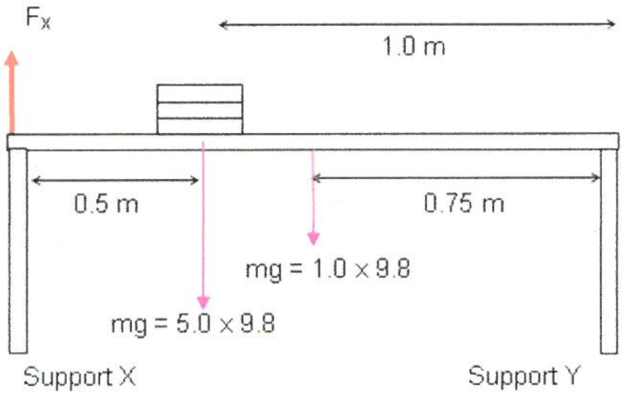


The total upward force has to balance the total (downward) weight.

F = (5+1) x 9.8 = 58.8 N

∴ **59 N (ANS)**

Question 13 1999 Question 4, 43%



Take torques about support Y.

$F_X \times 1.5 = 9.8 \times 0.75 + 49 \times 1.0$

$$\therefore F_X = \frac{9.8 \times 0.75 + 49 \times 1.0}{1.5}$$

$$\therefore F_X = \frac{56.35}{1.5}$$

$\therefore F_X = 37.6\text{N}$

∴ 38N (ANS)

Question 14 1997 Question 11, 52%

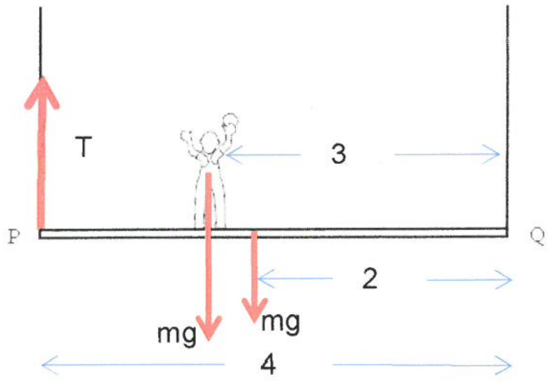
As the cleaner moves from P to Q, the tension in P will decrease and the tension in P will increase. (You can think this out by pretending that the cleaner goes all the way to Q).

The combined mass of the cleaner and the wooden plank is 60 + 10 = 70 kg. ∴ the weight force is 700N, ∴ the sum of the two tensions must be 700N

∴ B (ANS)

Question 15 1997 Question 12, 30%

Take moments about Q.



∴ T x 4 = mg x 3 + mg x 2

∴ 4T = 600 x 3 + 100 x 2

∴ 4T = 2000

∴ **T = 500 N (ANS)**

Question 16 1996 Question 1, 61%

The question asks for the contact forces **on** the board, (25% of students got the directions incorrect on the exam). To balance the weight of the board and Bob, P must push down on the board and Q must push up on the board.

Question 17 1996 Question 2, 26%

Take the torques around Q.

Let R = weight of diving board and

B = weight of Bob

∴ P x 1.0 = R x 1.5 + B x 4

∴ P = 40 x 10 x 1.5 + 80 x 10 x 4

∴ P = 600 + 3200

∴ **P = 3800 N (ANS)**

Question 18 1996 Question 3, 57%

As Bob walks from S to R, (because he wimped out on taking the dive) Then the bending moment about Q from his weight must decrease. This means that the force at P will also decrease. This means that the total downward force has decreased, therefore to maintain equilibrium, the force at Q must also decrease

$\therefore$ **B (ANS)**

Question 19 2016 Question 11, 64%

Torques equation about pivotal point C:

$0.05 \times g \times 0.3 + 0.2 \times g \times 0.15 = T \sin 30° \times 0.3$

T = 2.94 N (ANS)

Question 20 2015 Question 11, 56%

Torques equation about pivotal point M:

$40g \times \sin 30° \times 10 + 20g \times \sin 30° \times 5 = T \times 7$

T = 350 N (ANS)

Question 21 2014 Question 7, 56%

Torques equation about pivotal point X:

$0.05g \times 0.4 = T \sin 30°$

T = 0.98 N (ANS)

Question 22 2014 Question 8, 39%

The sum of the torques about any point on the beam must be zero. Taking torques about point W, the weight of the beam and the tension in the string have no effect because they act through the point about which torque is being evaluated. Therefore, the torque caused by the force of the wall on the beam must also be zero. This will only occur if this force also acts through the point W, which would be directly along the beam. There must be a non-zero force to balance the horizontal component of the tension in WZ.

Direction B (ANS)

Question 23 2014 Question 9, 60%

$\tau = 0.15g \times 0.8 = 1.176\ Nm$

1.2 Nm (ANS)

Question 24 2013 Question 6, 60%

$\tau = 20g \times \cos 60° = 588\ Nm$

588 Nm (ANS)

Question 25 2013 Question 7, 32%

Torques equation about pivotal point P: (this time it is more convenient instead of finding resolution of the force perpendicular to the lever arm use perpendicular distance to the line of the force application)

$588 + 10g \times 1.5 = T \times 4$

T = 183.75 N (ANS)

Question 26 2013 Question 8, 42%

Torques equation about pivotal point K: (this time it is more convenient instead of finding resolution of the force perpendicular to the lever arm use perpendicular distance to the line of the force application). We will find vertical reaction force from the support on the beam and according to the 3rd Newton's law force exerted by the beam on support will be equal in the magnitude.

$1000g \times 6 + 4000g \times 3 = N \times 2$

N = 88200 N (ANS)

Question 27 2012 Question 8, 51%

Torques equation about pivotal point 'securing bolt':

$12000g \times 30 = N \times 20$

N = 176400 N (ANS)

Question 28 2012 Question 9, 58%

We will use the fact that net force equal to 0, so sum of the forces up will be equal to the sum of the forces down.

$F + 12000g = 176400$

N = 58800 N (ANS)

Question 29 2011 Question 8, 64%

Torques equation about pivotal point A: (this time it is more convenient instead of finding resolution of the force perpendicular to the lever arm use perpendicular distance to the line of the force application)

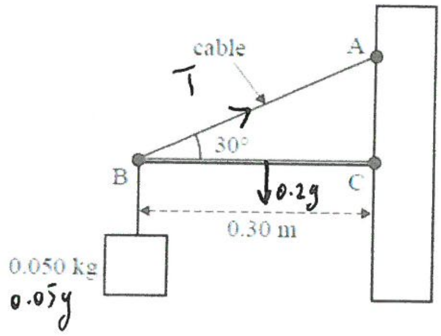
$40g \times 2 = T \times 2$

T = 396 N (ANS)

Rotational equilibrium questions

Question 19 2016 Question 11 64 %

A horizontal beam, BC, is attached to a wall, as shown. The mass of the beam is 0.20 kg and there is a hanging mass of 0.050 kg at point B. The point C is a hinge. Ignore the mass of the cable AB.



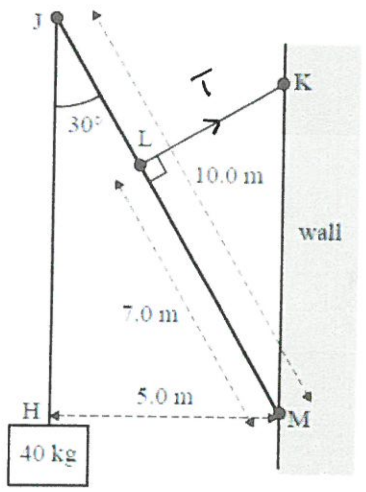
$0.05g \times 0.3 + 0.2g \times 0.15 = T \sin 30° \times 0.3$

$T = 2.94$

Find the tension in the cable AB.

Question 20 2015 Question 11 56 %

A uniform beam, JM, of mass 20 kg and length 10 m is joined to a wall at point M by a frictionless hinge. A cable, KL, connects the beam to the wall, as shown. Ignore the mass of the cable. A mass of 40 kg hangs from point J with a cable JH.



$40g \times 10 \times \sin 30° + 20g \times 5 \sin 30° = T \times 7$

$200g + 50g = 7T$

$T = \frac{250g}{7} = 350 \text{ N}$

Find the tension in KL.

Rotational equilibrium questions

Use the following information to answer Questions 21–23.

In an experiment, students set up the cantilever structure shown in Figure 3a below and attach it to the wall PQ. The pivot at X is a freely rotating hinge. The mass of the beam XY is 50 g and the string ZW can be considered to have no mass.

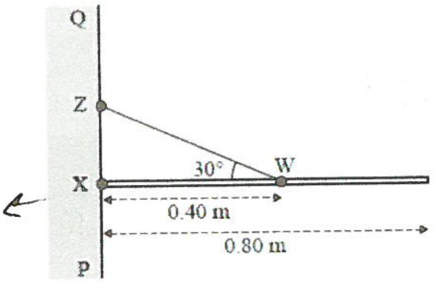


Figure 3a

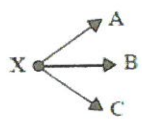


Figure 3b

Question 21 2014 Question 7 56%

Find the tension in the string ZW.

$T \sin 30° \times 0.4 = 0.05g \times 0.4$

$T = \frac{0.05g}{\sin 30°} = 0.98\ N$

Question 22 2014 Question 8 39%

Using Figure 3b as a direction reference, which one of the following best describes the direction of the force of the wall PQ on the beam XY at point X?

Question 23 2014 Question 9 60%

A 150 g hanging mass is now attached at point Y, as shown in Figure 4.

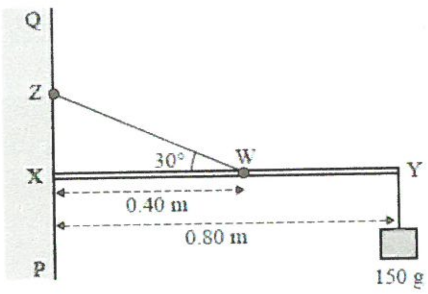


Figure 4

Find the torque on the beam about point X due to the 150 g hanging mass.

$0.15g \times 0.8$ 1.2

Rotational equilibrium questions

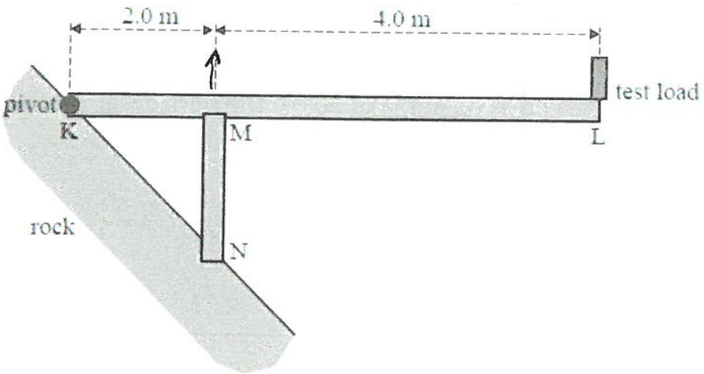


Figure 3

Find the force exerted by the beam KL downwards on the support MN.

$1000g \times 6 + 4000g \times 3 = N \times 2$

$18000g = 2N$

$N = 9000g$ 88200

Use the following information to answer Questions 27 and 28.

A new theatre has an entrance with a cantilever roof protruding from the main structure. The cantilever roof has a mass of 12 000 kg and is attached by a securing bolt to an anchor structure that is embedded firmly in the ground. The cantilever roof has uniform thickness and density, and is also supported (as shown) 40 m from its free end. Figure 4 shows the situation from the side.

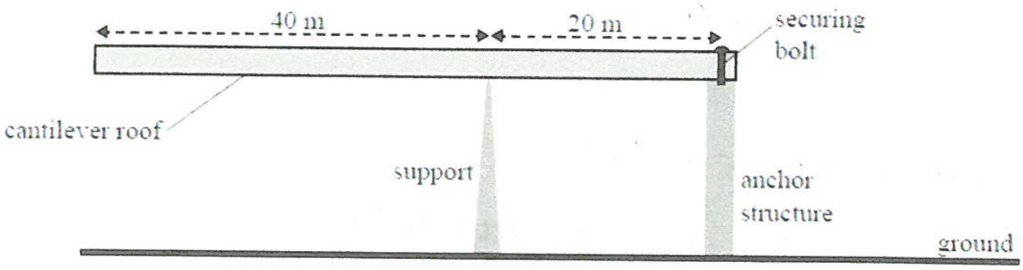


Figure 4

Question 27 2012 Question 8 51 %

Find the magnitude of the force that the support exerts on the cantilever roof.

$12000g \times 30 = N \times 20$

$N = 18000g$ 176 400

Question 28 2012 Question 9 58 %

Find the force with which the anchor structure is pulling downward on the cantilever roof

$-12000g = N_1 + N_2$

$N_2 = -120000g + 176400$ ~~294 000~~ ~~999600~~ 58800

Rotational equilibrium questions

Use the following information to answer Questions 24 and 25.

A rod PQ of mass 10 kg and length 6.0 m is joined to a wall at point P by a frictionless hinge. A mass of 20 kg hangs from point Q. A cable SR holds the rod at an angle of 60° to the horizontal. SR makes an angle of 90° with the rod PQ. The situation is shown in Figure 2.

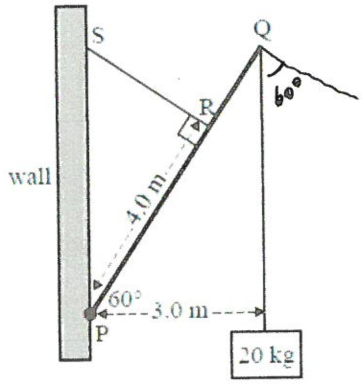


Figure 2

Question 24 2013 Question 6 60 %

Find the torque about point P due to the 20 kg mass.

$20 g \cos 60° \times \quad 20 g \times 3 = 60 g \quad 588\ Nm$

Question 25 2013 Question 7 32 %

Find the tension in the cable SR.

$$588 + 10g \times 1.5 = T \times 4$$

$$T = \frac{588 + 15g}{4} = 183.75$$

Question 26 2013 Question 8 42 %

A platform is constructed with a concrete beam KL of length 6.0 m and mass 4000 kg. The concrete beam is reinforced with steel rods. It is attached to stable rock at point K with a frictionless pivot and supported by a column MN. M is located 2.0 m from point K. The situation is shown in Figure 3. The designers test the platform with a test load of 1000 kg, located at the end of the platform at point L.

Rotational equilibrium questions

A school crossing sign is supported by a rigid rod, AC, smoothly hinged to an upright pole at point A, and a cable, BC, as shown in Figure 3.

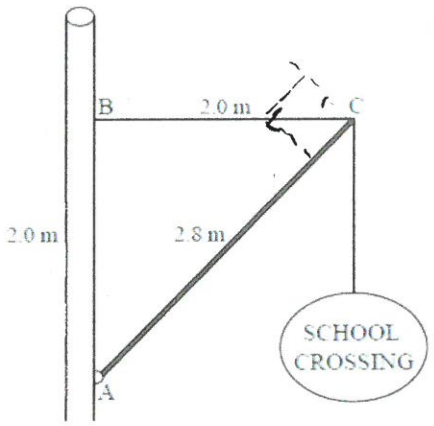


Figure 3

Length of rod AC = 2.8 m

Length of cable BC = 2.0 m

Length AB = 2.0 m

Mass of sign = 40 kg

Ignore mass of rod and all cables.

Question 29 2011 Question ~~7~~ 8 64%

Find the magnitude of the force in the cable BC.

$40g \times 2 = T \sin 45^\circ \times 2.8$

$T = \frac{80g}{2.8 \sin 45^\circ} = \frac{80g\sqrt{2}}{2.8} = 400.74$

396 N (1)31