Relativity

Study Design

• describe Einstein's two postulates for his theory of special relativity that:

–the laws of physics are the same in all inertial (non-accelerated) frames of reference

–the speed of light has a constant value for all observers regardless of their motion or the motion of the source

• compare Einstein's theory of special relativity with the principles of classical physics

• describe proper time (t_0) as the time interval between two events in a reference frame where the two events occur at the same point in space

• describe proper length (L_0) as the length that is measured in the frame of reference in which objects are at rest

• model mathematically time dilation and length contraction at speeds approaching c using the equations: $t = t_0\gamma$ and $L = \frac{L_0}{\gamma}$ = where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

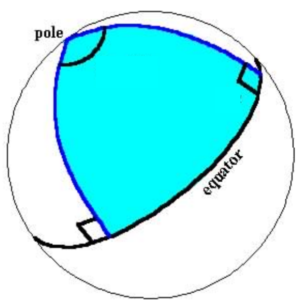
• explain why muons can reach Earth even though their half-lives would suggest that they should decay in the outer atmosphere.

- **interpret Einstein's prediction by showing that the total 'mass-energy' of an object is given by: $E_{tot} = E_k + E_0 = \gamma mc^2$ where $E_0 = mc^2$, and where kinetic energy can be calculated by: $E_k = (\gamma - 1)mc^2$**

- **describe how matter is converted to energy by nuclear fusion in the Sun, which leads to its mass decreasing and the emission of electromagnetic radiation.**

There was a young lady from Bright
Who travelled much faster than light
She departed one day
In a relative way
And returned on the previous night.

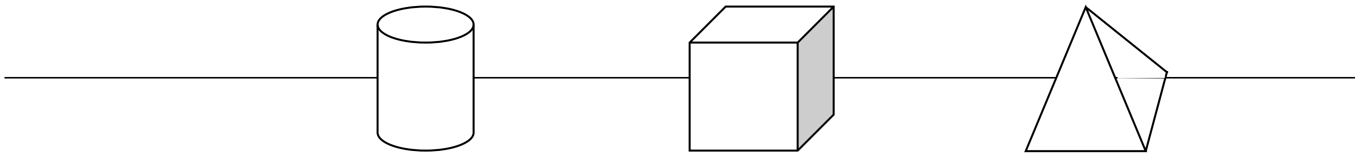
Let us start with a thought experiment to review some of our strongly held beliefs. The sum of the angles in a triangle add up to 180^0.
Is this always true?
Consider the following idea.
Horizontal lines of latitude are perpendicular to lines of longitude.
Look at the following diagram:
If we consider the triangle as shown, the sum of the interior angles is now greater than 180^0.
Therefore, the sum of the angles adding up to 180^0 is only true for plane (flat) surfaces.



Relativity is likely to cause us to suspend some of our beliefs, and to have our thinking challenged. Enjoy!

Location

Where are you? Such a simple question has a very complex answer. Two people can give very different answers to the same question, even if they are apparently in the same position.



If I say that the cube is 5 cm right, I need to give a reference point, such as 5 cm right of the cylinder. This means that for an answer to be the question of "Where are you?" to make sense, we must give a reference point. In physics, we define **frames of reference** for every measurement. If I say the cube is "5 cm left of the pyramid", that answer is just as correct as the answer "5 cm right of the cylinder". In other words, every frame of reference is equally valid for making statements; **there is no preferred frame of reference**.

Frames of reference

Frames of reference are objects or co-ordinate systems with respect to which we take measurements.

We need to consider the frame of reference when we solve all physics problems. Imagine yourself in a soundproof box. You can't see out of the box. If you cannot see out of the box, how could you tell if the box was moving? It is simple if the box is accelerating – you will feel that acceleration. But if the box is travelling at a constant speed or stationary, there would no way to tell from inside. In a box moving at a constant speed (nb. A stationary box is moving at a constant speed of v = 0) an object you dropped would fall directly to the floor, but if the box were accelerating, it would appear to fall backwards in the frame of reference of the box.

These examples give us our two types of reference frames:

1. *Inertial Frames of Reference: moving at a constant speed or stationary*
2. *Non-inertial Frames of Reference: accelerating or decelerating*

We will work exclusively with inertial frames of reference, but you must be able to identify non-inertial frames. One of the more common examples is when moving in circular motion.

Inertial frames

Inertial frames of reference are where the frame is not accelerating, therefore they are either at 'rest' or moving with constant velocity with respect to other *inertial frames of reference.* No frame is more correct than any other, but some are simpler. **Newton's first law is obeyed in all Inertial frames of reference**.

> We can assume that the ground is an inertial frame of reference provided that the distances we are using allow us to neglect the actual astronomical motions.

Newtonian relativity

No mechanical experiment done within an inertial frame of reference can distinguish between rest and constant velocity. The results for all mechanical experiments in either a rest frame of reference or a constantly moving frame of reference will always be identical.

If you do an experiment in class, or in a train moving at constant velocity, you could not tell from the experiment whether you were at rest or in the moving train.

> Eg. If you drop a ball in a classroom, it will land vertically under the point of release.
>
> The same thing will happen in a train moving

> The implication from this is that distance (space) and time are the same in all inertial frames of reference.

Relative motion in one dimension

You are on a train, travelling north at 60 km h^{-1}. To another passenger (1st observer) on the train, you seem to be stationary. To a person (2nd observer) standing on the platform (as the train goes by) you appear to be travelling north at 60 km h^{1}.

Your velocity depends on the frame of reference of whoever is observing or measuring the velocity.

A reference frame is the physical object to which we attach our co-ordinate system. In everyday life, that object is the ground.

Eg. A body moving with velocity 10 m s^{-1} East would, to a stationary observer, appear to be moving east at 10 m s^{-1}. If the observer was also moving with velocity 10 m s^{-1} East, then the object would appear to be stationary. If the observer was moving in any other direction, then $\text{velocity}_{\text{A relative to B}} = \mathbf{v}_{ab} = \mathbf{v}_a - \mathbf{v}_b$.

In general if a body *A* and an observer *B* are moving with velocities $\boldsymbol{v}_a$ and $\boldsymbol{v}_b$ respectively, then the velocity of body *A* relative to observer *B* ($\boldsymbol{v}_{ab}$) is the velocity the body *A* appears to have to observer *B*, when *B* is unaware of their own motion, i.e., when the observer has apparent zero velocity. We require a frame of reference in which the observers velocity is zero. To obtain this we need to subtract the velocity of the observer.

$$\text{velocity}_{\text{A relative to B}} = \boldsymbol{v}_{ab} = \boldsymbol{v}_a - \boldsymbol{v}_b.$$

Applying this concept to the train example above.

For observer 1 (on the train) $v_{ab} = v_a - v_b$, since $v_b = v_a \therefore v_{ab} = 0$.

$\therefore$ You appear stationary.

For observer 2 (on the platform) $v_{ab} = v_a - v_b$, since $v_b = 0 \therefore v_{ab} = v_a$

$\therefore$ You appear to be travelling north at 60 km h^{-1}

Motion in two dimensions

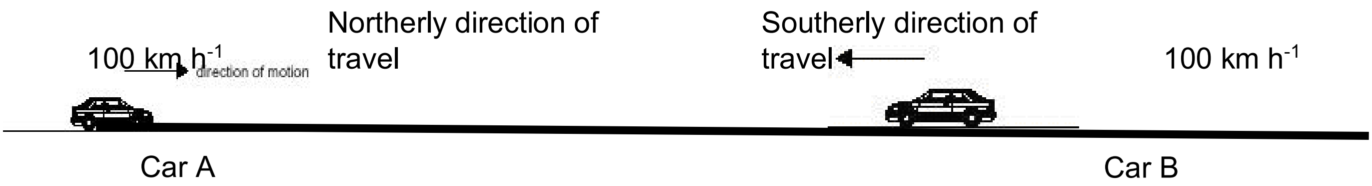
Our two observers are again watching you on the train, this time you are walking across the train (easterly) as the train is travelling in a northerly direction.

For observer 1 (on the train) you appear to travel in an easterly direction.

For observer 2 (on the platform) you appear to be travelling in a north-easterly direction.

Distinguish between stationary frames of reference and frames of reference that are moving at constant speed relative to the stationary frame.

Consider two cars moving towards each other in opposite directions at 100 km h^{-1} on a freeway. The stationary frame of reference is the ground. You are an observer in a moving frame of reference, car A.



To an observer in the stationary frame of reference (on the ground),

Car A is travelling North at 100 km h^{-1}.

Car B is travelling South at 100 km h^{-1}.

It can be shown in all cases that, observers on different frames of reference (that move with a constant velocity relative to each other) will measure the same acceleration for body *A*. Therefore they measure the same forces acting on the body.

As for one dimensional motion, we have the following rule: Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration (and force) for a moving particle.

When we measure velocity it is very important to take frames of reference into account. If you throw a ball at a speed of 10 m s^{-1}, you are automatically defining your frame of reference with you as stationary.

If you threw the same ball, with the same speed, while standing on the back of a truck, you must take into account the speed of the truck and also the direction in which you throw the ball.

Consider the issue of a person standing still on the roadside. What speed will the observer see the ball travel at? The truck travels forwards at 40 m s^{-1}, the ball backwards (*relative to the truck*) at 10 m s^{-1}, so we add the velocities (remember vector addition!):
= 40 m s^{-1} forwards + 10 m s^{-1} backwards
= 30 m s^{-1} forwards.

So the ball travels at a velocity less than the truck, in the same direction of the truck, according to the observer at the roadside. According to the thrower on top of the truck, the ball is travelling backwards at 10 m s^{-1}. Both are correct, but each is in a different frame of reference.

Galilean relativity and Maxwell's prediction about the velocity of light

Maxwell predicted that the velocity of light in a vacuum would be equal to *c* (relative to the ether). Hence observers moving relative to the ether should observe a change in speed of light (due to Galilean relativity).
Eg. If the source of light is moving through the ether at a speed of *v* (in the same direction as the light), then the speed of light relative to the source should be *c – v*. If it moves in the opposite direction (ie with the ether) then the speed of light relative to the source will be *c + v.*

Historical Development of the theory

Motion

Galileo determines that all inertial frames of reference are equally valid.

Hence velocity measurements depend on the frame of reference they are measured in.

Newton uses Galileo's ideas and develops his laws of motion.

All frames of spatial reference are equally valid, but in Newton's mechanics, time is absolute.

Hence whatever frame you are in you would measure the same time between two events

Light

Romer measures the speed of light by observing the moons of Jupiter.

Using his equations of electromagnetism, ***Maxwell*** discovers that light is an EM wave that always travels at a constant speed *c*.

Maxwell proposes that this speed *c* is relative to the ether, and hence there is an absolute frame of reference

Michaelson and Morley perform a precise experiment to detect the presence of an ether, but find no evidence of its existence.

Einstein proposes that
1) All frames of reference are equally valid
2) The speed of light is a constant, *c*, measured in any frame of reference.

Einstein agrees with Galileo/Newton that all spatial frames of reference are equally valid, but in Einstein's relativity time is no longer the same in all frames.

Einstein agrees with Maxwell that light always travels with a constant speed, *c*, but he does away with the need for the ether. There is no absolute spatial frame

Time-Space-Motion

Time is measured by motion in space, be it by the motion of the earth around the sun (the year), the earth on its axis (the day) or the vibration of the Caesium - 133 atom. (the SI definition of the second). But motion itself is defined by space and time - the specification of changes in position with respect to time. Space, itself, can be defined by time and motion.

That is, time, space and motion are only measurable with respect to one another.

Newton defined how, and under what conditions, motion changes with time. Einstein showed how time changes with motion.

Newton's "theory of relativity" stated that the **laws of mechanics are the same in all inertial frames of reference** ie. all frames of reference which are moving with constant velocity with respect to each other.

ie Uniform motion is relative - not absolute.

(Newton, however, could not accept that time was anything other than absolute).

Maxwell and light

Maxwell (~1862) proposed a newer model for light and gave a mathematical definition for the electromagnetic wave model. He integrated the laws of electricity and magnetism by giving mathematical evidence for a model that had interacting electric and magnetic fields, each field perpendicular to the other.

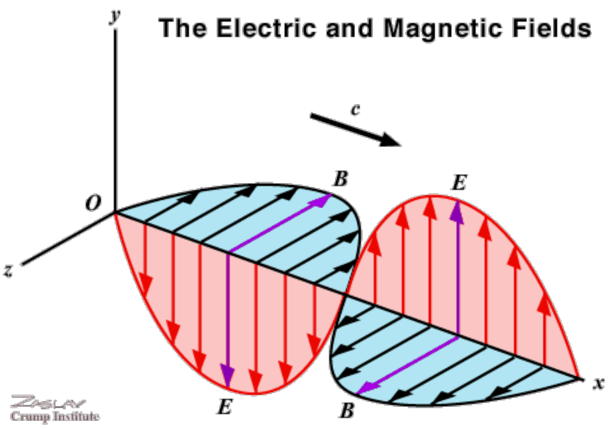
Maxwell used the following ideas:

Electric charges make electric fields
Moving electric charges make magnetic fields
A changing electric field makes a magnetic field
A changing magnetic field makes an electric field

Maxwell combined these ideas to develop a model for an electromagnetic wave, consisting of perpendicular oscillating electric and magnetic fields.

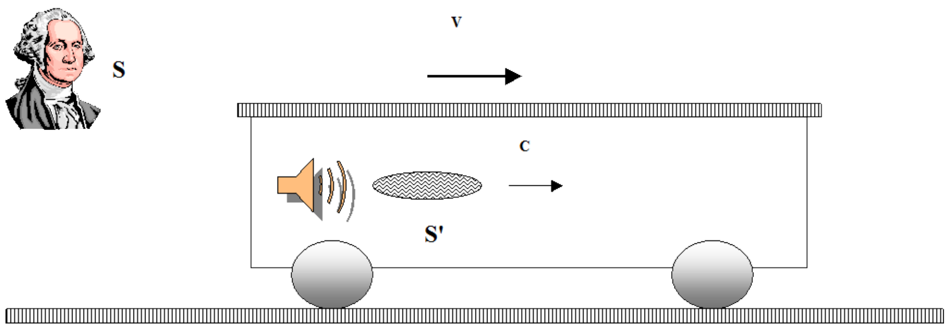
The propagation speed of these oscillations is determined by the universal constants of electricity and magnetism.

He described these waves as electromagnetic waves. Romer (1675) and Bradley (1728) had measured the speed of light and their results were consistent with Maxwell's.



Galilean transformations and the speed of light

Consider a light pulse sent out by observer S^1 in a railway boxcar moving with a speed v with respect to a stationary observer S.



The observer S would measure the speed of the light pulse as v + c.

But classical electromagnetic theory (Maxwell) said that the speed of light was always a fixed value (3×10^8 m s^{-1} in vacuum regardless of the motion of the observer).

∴ Either

1. Galilean transformations are incorrect or
2. The laws of electricity and magnetism are not the same in all inertial frames. That is, they change according to the motion of the observer.

The speed of light

The speed of light was predicted to be ~ 300,000 kilometres per second, 3×10^8 m s^{-1}.

More recent calculations have led to the figure of 299 792 458 m s^{-1}, which is now a defined quantity. Indeed, the standard unit of a meter has been defined as $\frac{1}{299\,792\,458}$ of the speed of light.

The symbol ***c*** is used for the speed of light. ***c*** stands for the Latin word *celeritas*, meaning "fast", which is appropriate since the speed of light is the "speed limit" for the universe – nothing may travel faster than it.

$$c = 299\,792\,458 \text{ m s}^{-1}$$

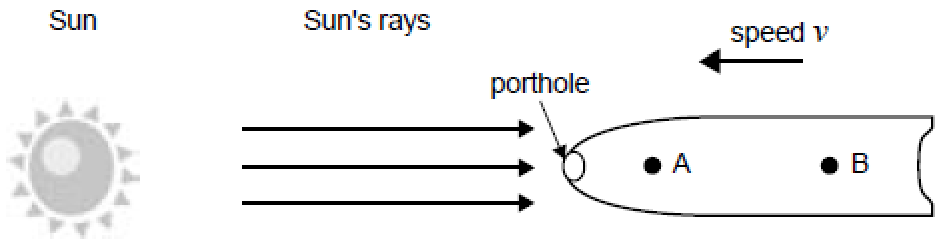
Maxwell's electromagnetic model of light caused some problems, particularly the idea of an absolute speed of light. An absolute speed of light – the idea that no matter the source, any observer will record the same speed, regardless of relative position. This problem would not be resolved until Einstein's Theory of Special Relativity (1905).

The second problem was with the idea of an electromagnetic wave. All other waves have to travel through a medium – but light was known to travel through the vacuum of space. Maxwell proposed that space was not a vacuum, but a "massless, rigid medium" Maxwell named **Æther**.

This material was invisible and did not affect the movement of physical objects through it (i.e. the planets moving through "empty" space). Maxwell believed that the speed of light was only constant with respect to the ether. In other words, the motionless ether formed the absolute frame of reference.

Imagine two students travelling in a spaceship toward the Sun at speed *v* (see below). They plan to measure the speed of light in two experiments, as a test of the prediction of James Clerk Maxwell and that of Galilean relativity.

In the first experiment, they determine the speed of light (*c*) within the rocket ship by measuring the time for a short pulse of light emitted from a flashbulb at point A to reach point B. They got the accepted value for *c*. In the second experiment they determined the speed of light for a beam of light from the Sun, which passed through a porthole in the rocket nose. This was done by measuring the time for the light to travel between point A and point B.



EINSTEIN'S THEORY

The fact that light seemed to have a constant velocity, and is self-propagating (travels as a wave without a medium) was at odds with traditional Newtonian models and classical physics understanding of velocity addition.

Take the following simple thought experiment:
A spaceship travelling at half the speed of light fires a laser forwards directly along its flight path. What is the speed of the laser as measured by the crew?
By a stationary astronaut floating in space?

It would seem that there are two answers – one for the crew (c) and one for the astronaut (1.5c).

This creates two problems – are two answers acceptable, and is light in this circumstance travelling faster than the speed of light?

In 1905, Einstein considered these questions and proposed his theory of special relativity to resolve them. His theory is two main ideas, called postulates.

EP1**The Principle of Relativity**
All the laws of physics are the same in all uniformly moving (inertial) frames of reference. (This compares with Newton's assumptions that the laws of mechanics are the same in all inertial frames)

EP2**The Constancy of the Speed of Light**
The speed of light in vacuum is the same (3×10^8 m s^{-1}) in all inertial frames (ie there is no ether) and the speed of light is the same regardless of the motion and the source of light.

The first postulate is critical, it states that anything occurring in any reference frame can be explained by the rules of physics that apply on Earth.

The second postulate implies a "universal speed limit" – nothing may travel faster than the speed of light. Even light itself, when emitted from a travelling object still travels at the speed of light.

Looking back at the example of the spaceship and the laser above, it means that the laser speed will be c – meaning that the spaceships own velocity has no effect on the ending velocity of the laser. The law of addition of velocities seems not to apply in this circumstance!