Projectile Motion

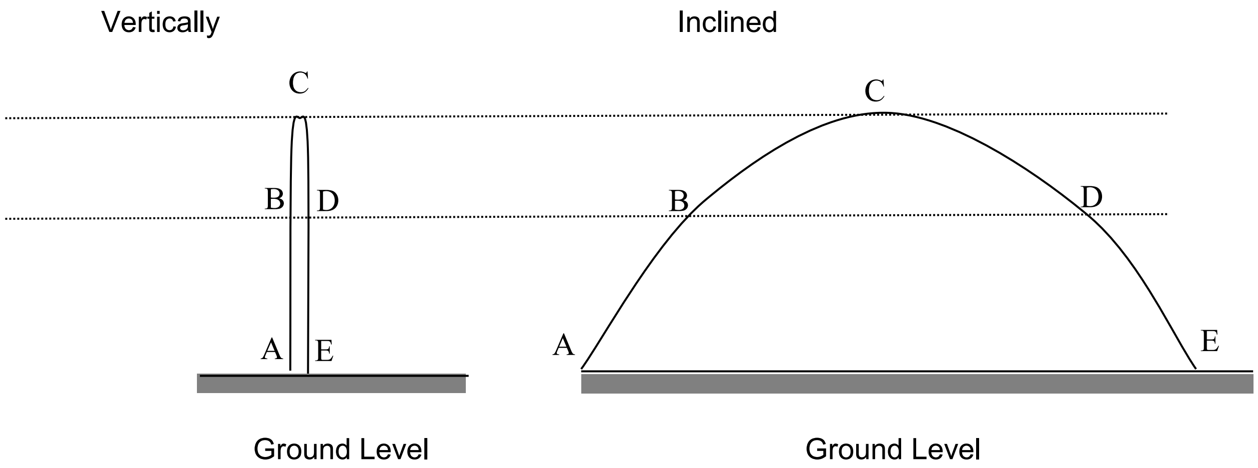
Study Design

- **investigate and analyse theoretically and practically the motion of projectiles near Earth's surface, including a qualitative description of the effects of air resistance**

Introduction

A projectile is a body that has been thrown or projected, and is travelling freely through the air. While undergoing projectile motion the object is under the constant unbalanced force of gravity. When we study projectile motion, no consideration is given to the force projecting the body or to what happens when it lands. Air resistance is considered to be negligible (in quantitative questions), but can be taken into consideration in qualitative questions. The projectile will travel either: vertically or inclined, depending on the initial angle of projection.

The vertical and horizontal motions are treated independently. Usually the vertical motion is treated first, since it determines how long the projectile is in the air.



For both the vertical and inclined projectiles:

- the only force acting is the weight, ie. the bodies are in free fall
- acceleration is **always** 9.8 ms^{-2} downward, (including the point C)
- the instantaneous velocity is tangential to the path
- the total energy (KE & PE) is constant
- between any two points $\Delta KE = - \Delta PE$
- paths are symmetrical for time eg. Δt (A to B) = Δt (D to E):
  Δt (A to C) = Δt (C to E)
- paths are symmetrical for speed eg. speed at A = speed at E; speed at B = speed at D.
- for vertical motion $v_c = 0$, for inclined motion $v_c \neq 0$.

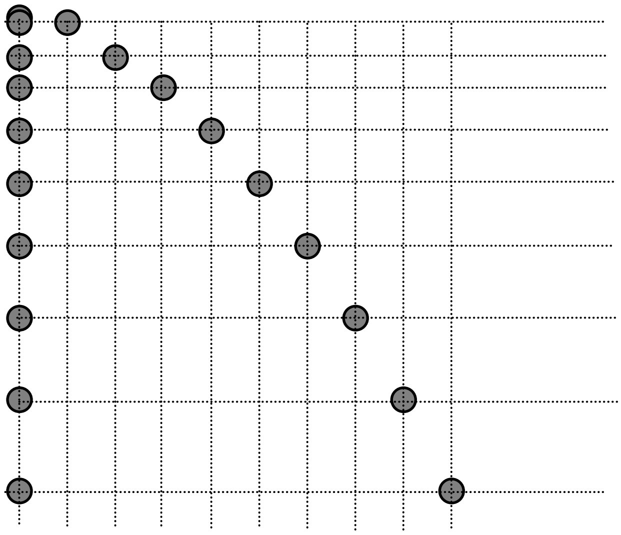
Inclined or oblique projections

- the only force acting is vertically down, so the acceleration and change in velocity are vertical.
- horizontally there is no component of force, so constant horizontal velocity.
- Maximum range is when angle of projection is 45^0

Horizontal projection

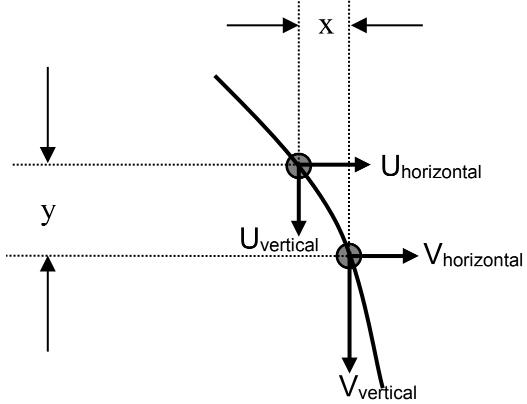
When the body is launched horizontally and follows a parabolic path to the ground it is really the second half of an inclined projection. The time of flight is controlled by the height from which it is released. The speed of projection will not affect the time 't' that it takes to land. The 'range' of this projectile is given by the $x = v_{horizontal} \times t$.

For projectiles thrown horizontally and dropped from rest, the vertical motions are the same. This can be shown by a multi-flash photograph.



The interval between the lines represents how far the ball has travelled in a small time interval. Notice that the distance between the horizontal lines increases as the ball descends, indicating that the ball is speeding up. The distance between successive vertical lines remains constant, indicating that the ball is travelling with a constant velocity in the horizontal direction.

If we analyse the motion by using resolution of vectors we get the following:



Horizontal:

velocity always = $v_{horizontal}$
acceleration = 0
displacement = $x = v_{horizontal} \times t$

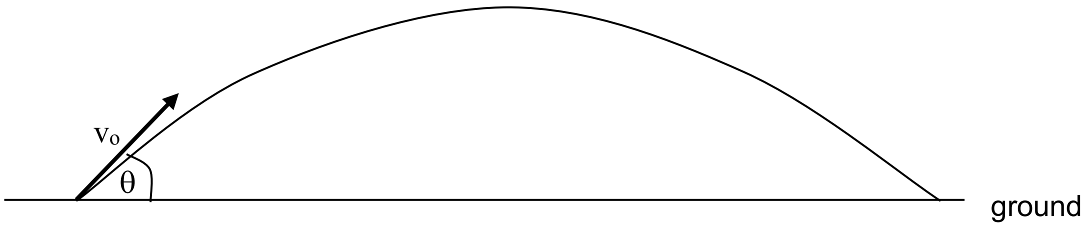
Vertical:

Velocity $v = u + gt$
acceleration = g
displacement $y = ut + ½ gt^2$

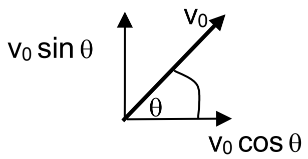
To find the 'total' velocity, add $v_{vertical}$ and $v_{horizontal}$ using vectors.

If one projectile was fired horizontally, at the same time that another was dropped (from the same height), then both objects would hit the ground at the same time. This is because both their vertical motions were identical. (Same distance to fall, initial speed = 0, and acceleration = -g)

Inclined Projectiles.



The vector representing the initial velocity can be resolved into two components



Horizontal: velocity always = $v_{horizontal}$

$v_{horizontal} = v_0 \cos\theta$

acceleration = 0

displacement = $x = v_0 \cos\theta \times t$

Vertical (on the way up) velocity changing $v = u - gt$

$V_{vertical} = v_0 \sin\theta - gt$

acceleration = -g

Vertical (on the way down) velocity changing $v = u + gt$

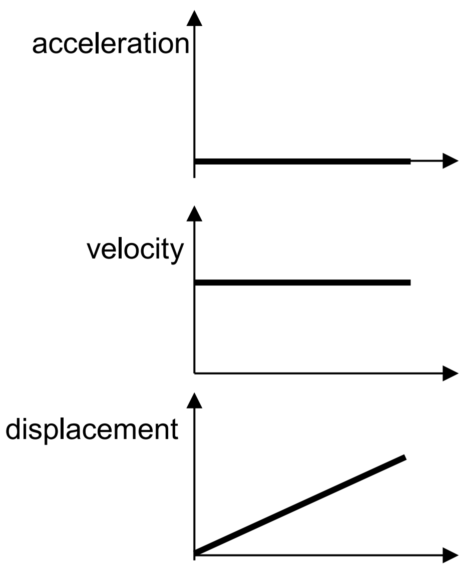
$v = 0 + gt$

acceleration = g

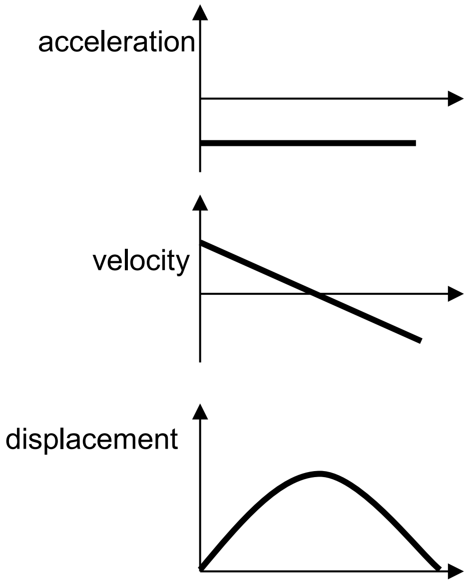
The displacement at any time of the motion is: $y = ut - \frac{1}{2} gt^2$ (The same on the way up and down).

Graphs for projectile motion

motion in the 'x' direction (right is positive)



motion in the 'y' direction (up is positive)



Symmetrical flights

If there is no air resistance, and the projectile starts and ends at the same height, then the range is given by: $R = \dfrac{v^2 \sin 2\theta}{g}$ R is the range, v is the initial speed and θ the angle of projection. Be careful using this formula, because it only works under the conditions specified above.

Total Energy (TE)

- If air resistance is negligible, then the total energy of a projectile will always remain constant throughout the flight. TE = KE + PE = $\frac{1}{2}mv^2 + mgh$.
- At ground level PE = 0, so TE = KE.
- As the projectile rises it gains PE, so it must lose KE. At the top of its flight, the PE is maximum and the KE is minimum. (the KE is not zero, because the projectile still has some KE due to its horizontal motion).
- At any point on the way up or the way down, the TE is constant.
- If you know the horizontal component of the velocity, then you can use this to find the maximum height.
- Use the TE at ground level and work out what the PE must be at the top when $v_{vertical} = 0$, but $v_{horizontal}$ = constant.