Past VCAA questions Wave properties of matter solutions

Solutions
Multiple choice solutions
Example 1 2007 Question 6, 47%

Use $x = \frac{\lambda L}{d}$. As d increase, spread decrease.

$\therefore$ **B (ANS)**

Example 2 2005 Question 8, 71%

Photons of light have momentum and energy

$\therefore$ **B, D (ANS)**

Example 3 2004 Question 7, 68%

Diffraction patterns only depend on wavelength.
The diffraction patterns for the electrons and x-rays were almost identical, therefore they had the same wavelength.

$\therefore$ **A (ANS)**

Example 4 2002 Question 4, 59%

The diffraction pattern is due to the wavelike properties of each.

B (ANS)

Example 5 1995 Question 6, 70%

Diffraction patterns are produced by waves. Hence, both X-rays and electrons exhibit wave-like properties when they pass through the foil.

$\therefore$ **B (ANS)**

Example 6 1972 Question 60, 25%

The resolution is impacted by the dispersion. The dispersion is given by $\frac{\lambda}{d}$, so to decrease the dispersion decrease λ and/or increase d.

$\therefore$ **B, C (ANS)**

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Short answer solutions

Example 7 2008 Question 9, 70%

Use $\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.0 \times 10^{7}}$

$= 3.64 \times 10^{-11}$

$= 3.6 \times 10^{-2} \times 10^{-9}$

= 3.6×10^{-2} nm (ANS)

Example 8 2007 Question 4, 60%

The de Bröglie wavelength is given by

$\lambda = \frac{h}{mv}$

$\therefore v = \frac{h}{m\lambda}$

$= \frac{6.6 \times 10^{-34}}{1.67 \times 10^{-27} \times 2.0 \times 10^{-10}}$

$\therefore$ **2.0×10^{3} m/s (ANS)**

Example 9 2005 Question 6, 71%

The momentum of the electron

$= m \times v$

$= 9.1 \times 10^{-31} \times 2.0 \times 10^{7}$

$= 1.82 \times 10^{-23}$

Use $p = \frac{h}{\lambda}$

$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1.82 \times 10^{-23}}$

$\therefore$ **3.64×10^{-11}m (ANS)**

Example 10 2005 Question 7, 71%

There is diffraction when $\frac{\lambda}{w} \sim 1$.

In this case $\frac{\lambda}{w} = \frac{3.64 \times 10^{-11}}{5 \times 10^{-4}} \sim 10^{-7}$.

There will be no diffraction as $\frac{\lambda}{w} << 1$

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Example 11 2004 Question 8, 68%

From $p = \frac{h}{\lambda}$ where $\lambda = 35 \times 10^{-12}$ m, because 35 pm is 35 pico-metres.

$\therefore\ p = \frac{6.63 \times 10^{-34}}{35 \times 10^{-12}}$

$\therefore$ **1.9×10^{-23} kg m/s (ANS)**

Example 12 2004 Question 4, 85%

$E = \frac{V}{d}$

$\therefore\ E = \frac{100}{2 \times 10^{-2}}$

$\therefore$ **$E = 5.0 \times 10^{3}\ Vm^{-1}$ (ANS)**

Example 13 2004 Question 5, 43%

KE = Vq

$= 100 \text{ x } 1.6 \times 10^{-19}$

$= 1.6 \text{ x } 10^{-17}$.

$p = \sqrt{2m\,KE}$

$= \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-17}}$

$= 5.3963 \times 10^{-24}$

$\lambda = \frac{h}{p}$

$= \frac{6.63 \times 10^{-34}}{5.4 \times 10^{-24}}$

$\therefore$ **$1.23 \text{ x } 10^{-10}$m (ANS)**

Example 14 2003 Question 3, 45%

WD = ΔKE = qV

$\therefore\ \frac{1}{2} mv^2 = 1.6 \times 10^{-19} \times 2500$

$\therefore\ v^2 = \frac{2 \times 1.6 \times 10^{-19} \times 2500}{9.1 \times 10^{-31}}$

$= 8.79 \times 10^{14}$

$\therefore\ v = 2.96 \times 10^{7}$m/s.

This is close to 10% of 3×10^{8} m/s

Example 15 2002 Question 1, 44%

E = hf

$= \frac{hc}{\lambda}$

$\therefore\ \lambda = \frac{hc}{E}$

$= \frac{4.14\times10^{-15}\times3\times10^{8}}{70000}$

= 1.77 x 10^{-11} m (ANS)

Example 16 2002 Question 2, 44%

The separation of the lines in both diffraction patterns are identical, the wavelength of the electrons and X-rays must be the same (identical spacing of the diffraction patterns occur when the wavelengths are identical).

The de Broglie wavelength is

1.77 x 10^{-11} m (ANS)

Example 17 2002 Question 3, 16%

Find the momentum then use $E_k = \frac{p^2}{2m}$ to convert to energy.

de Broglie wavelength

$$\lambda = \frac{h}{p}$$
$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{1.77\times10^{-11}} = 3.75\times10^{-23}\,\text{Ns}$$

$$E_k = \frac{p^2}{2m} = \frac{(3.75\times10^{-23})^2}{2\times9.1\times10^{-31}} = 7.7\times10^{-16}\,\text{J} = \frac{7.7\times10^{-16}}{1.6\times10^{-19}}\,\text{eV} = 4818\,\text{eV}$$

So energy is **4.82 keV. (ANS)**

Example 18 2001 Question 3, 17%

The most difficult part to this question is identifying what it is asking. This is a question on diffraction comparing the effects of light and electrons.

The poorer quality of the optical image when compared with the image from the electron microscope is caused by the greater diffraction of the photons than the electrons. I.e. the sharper the image, the less the diffraction, as the rays are travelling in straight lines, so we get a sharp image.

Assume that the electrons are moving quite fast, giving them their longest wavelength, then typically the wavelength would be of the order of 10^{-11} m.

The wavelength of light is typically 10^{-7} m.

$\lambda_{electrons} << \lambda_{photons}$.

The scale of the diagram, shown in the bottom right hand corner is 25μm. This means that the wavelength of the light is similar to the width of sections of organism.

The amount of diffraction is given by the ratio of $\frac{\lambda}{d}$, so the electrons are much less likely to diffract, therefore this image will be clearer.

Example 19 2000 Question 7, 40%

Any two of the following

- the ring pattern was an interference/diffraction pattern formed by the X-rays passing through the metal
- the rings themselves represented positions of constructive interference
- the interference pattern occurs because the metal foil atoms/crystal planes act like a set of sources and a path difference results for the different scattered waves
- the pattern occurs because the wavelength of the X-rays is similar to that of the atomic spacing of the metal crystal.

Example 20 1999 Question 5, 70%

If KE = ½ mv^2 and p = mv, then KE = $\frac{p^2}{2m}$

p = $\sqrt{2m\,KE}$

$= \sqrt{2\times 9.1\times 10^{-31} \times 54 \times 1.6\times 10^{-19}}$

$= 3.97 \times 10^{-24}$ Ns

$\therefore$ **4.0 × 10^{-24} Ns (ANS)**

Example 21 1999 Question 6, 23%

For diffraction to occur, the ratio $\frac{\lambda}{w}$ must be approximately 1 – 50 λ

For the electrons $\frac{\lambda}{w} = \frac{1.67\times 10^{-10}}{2.15\times 10^{-10}}$ = 0.78

For 400nm light $\frac{\lambda}{w} = \frac{400\times 10^{-9}}{2.15\times 10^{-10}}$ = 1860.

$\therefore$ **the electrons will demonstrate interference.**

Example 22 1995 Question 4, 70%

From $\lambda = \frac{h}{p}$

We get $\lambda = \frac{6.63\times 10^{-34}}{7.94\times 10^{-25}}$

$\therefore$ **λ = 8.4 x 10^{-10}m (ANS)**

Example 23 1995 Question 5, 35%

The same diffraction pattern implies that the wavelength is the same.

The same wavelength implies that the momentum is the same.

$\therefore$**Momentum (ANS)**

Example 24 1993 Question 4

Use $p = \frac{E}{c}$

$\therefore$ $p = \frac{3.2 \times 10^{-15}}{3.0 \times 10^{8}}$

$\therefore$ **p = 1.1 x 10^{-23} Ns** **(ANS)**

Example 25 1993 Question 5

From $\lambda = \frac{h}{p}$

We get $\lambda = \frac{6.63 \times 10^{-34}}{2.06 \times 10^{-23}}$

$\therefore$ **λ = 3.2 x 10^{-11}m** **(ANS)**

Example 26 1993 Question 6

Momentum is conserved in all collisions.

P_{total} = 1.1 x 10^{-23}.

$\therefore$ $p_{photon\ after\ collision}$ = 0.96 x 10^{-23} (to the left)

From $\lambda = \frac{h}{p}$

Before the collision

we get $\lambda = \frac{6.63 \times 10^{-34}}{1.1 \times 10^{-23}}$

$\therefore$ λ = 6.02 x 10^{-11}m

After the collision

we get $\lambda = \frac{6.63 \times 10^{-34}}{0.96 \times 10^{-23}}$

$\therefore$ λ = 6.9 x 10^{-11}m

$\therefore$ **Increased** **(ANS)**

Another way to look at this is to just consider the equation $\lambda = \frac{h}{p}$, from this we can see that if the momentum of the photon has decreased (from 1.1 x 10^{-23} to 0.96 x 10^{-23}) then the wavelength must increase.

Example 27

The speeds are non-relativistic, so we can use p = mv and then $\Delta x \times \Delta p = \frac{h}{4\pi}$.

$\therefore$ $p = 9.11 \times 10^{-31} \times 2.05 \times 10^{6}$

$\therefore$ $p = 1.87 \times 10^{-24}$ kg m s^{-1}.

Uncertainty $\Delta p = 0.50\% \times p$

$\therefore$ $\Delta p = 0.0050 \times 1.87 \times 10^{-24}$

$\therefore$ $\Delta p = 9.35 \times 10^{-27}$

Use $\Delta x \times \Delta p = \frac{h}{4\pi}$.

$\therefore \Delta x = \frac{h}{4\pi \Delta p}$

$\therefore \Delta x = \frac{6.63 \times 10^{-34}}{4\pi \times 9.35 \times 10^{-27}}$

$\therefore \Delta x = 5.6 \times 10^{-8}$

$\therefore$ **Δx = 56 nm(ANS)**

Example 28

Uncertainty in $\Delta v = \frac{1.0}{20}$

$\therefore$ Uncertainty = 5%

Use p = mv

$\therefore p = 0.50 \times 20$

$\therefore p = 10$ kg m s^{-1}.

Uncertainty $\Delta p = 5.0\% \times p$

$\therefore \Delta p = 0.050 \times 10$

$\therefore \Delta p = 0.5$

Use $\Delta x \times \Delta p = \frac{h}{4\pi}$.

$\therefore \Delta x = \frac{h}{4\pi \Delta p}$

$\therefore \Delta x = \frac{0.6}{4\pi \times 0.5}$

$\therefore$ **Δx = 0.10 m (ANS)**

Example 29

Uncertainty in $\Delta v = \frac{1.0}{20}$

$\therefore$ Uncertainty = 5%

Use p = mv

$\therefore p = 0.50 \times 20$

$\therefore p = 10$ kg m s^{-1}.

Uncertainty $\Delta p = 5.0\% \times p$

$\therefore \Delta p = 0.050 \times 10$

$\therefore \Delta p = 0.5$

Use $\Delta x \times \Delta p = \frac{h}{4\pi}$.

$\therefore \Delta x = \frac{h}{4\pi \Delta p}$

$\therefore \Delta x = \frac{6.63 \times 10^{-34}}{4\pi \times 0.5}$

$\therefore$ **Δx = 1.1 × 10^{-34} m (ANS) Insignificant!**