Motion problems solutions.

1. Let distance from shore to shore is d.

   Time there $t_1 = \dfrac{S}{25}$, time back is $t_2 = \dfrac{S}{75}$

   Total time is $t_1 + t_2$

   $$\dfrac{S}{25} + \dfrac{S}{75} = 80 \qquad \dfrac{3S}{75} + \dfrac{S}{75} = 80 \qquad \dfrac{4S}{75} = 80$$

   $$S = \dfrac{75 \times 80}{4} = 1500 \, m$$

2. $t = \dfrac{1.5 \, km}{3 \, km\,h^{-1}} = 0.5 \, h$

3. $S = 70 \times 2 + 60 \times 1.25 = 215 \, km$

4. $S = (62 + 48) \times 6 = 660 \, km$

5. $S = (18 - 14) \times 1.25 = 5 \, km$

6. ~~Let speed of the boat is v, speed of the stream is u.~~

   Let distance upstream is S.

   Time upstream is $\dfrac{S}{4}$, time downstream is $\dfrac{S}{8}$.

   Total time is $\dfrac{S}{4} + \dfrac{S}{8} = 3 \qquad \dfrac{2S}{8} + \dfrac{S}{8} = 3 \qquad \dfrac{3S}{8} = 3 \qquad S = 8 \, km$

7. Let speed of walking is v. Speed of skiing will be $4v$. Time up is $\dfrac{3000}{v}$, time down is $\dfrac{3000}{4v}$.

   Total time is $\dfrac{3000}{v} + \dfrac{3000}{4v} = 20 \, min$

   $$\dfrac{12\,000 + 3000}{4v} = 20 \qquad v = \dfrac{15\,000}{4 \times 20} = 187.5 \, m\,min^{-1}$$

   $$4v = 187.5 \times 4 = 750 \, m\,min^{-1}$$

Let hilly distance is x

8. Time on level ground is $\frac{12-x}{10}$, time on hilly ground

$\frac{x}{6}$. Total time $\frac{12-x}{10} + \frac{x}{6} = \frac{5}{3}$ h

$\frac{3(12-x) + 5x}{30} = \frac{50}{30}$ $3(12-x) + 5x = 50$

                           $2x + 36 = 50$ $x = 7$ km

9. Let speed of the canoe relative to the water is v.

Time upstream is $\frac{12}{v-3}$, time downstream is $\frac{12}{v+3}$

$\frac{12}{v-3} - \frac{12}{v+3} = 2$ $\frac{12(v+3) - 12(v-3)}{(v-3)(v+3)} = 2$ $72 = 2(v^2 - 9)$

                                                $v^2 - 9 = 36$

$v^2 = 45$ $v = 3\sqrt{5} = 6.7$ km h^{-1}

Time in still water $t = \frac{12}{3\sqrt{5}} = 1.8$ h $= 1$ h 47 min

10. Let speed of the faster car is v. Time of faster

car is $\frac{300}{v}$. Time of the slower car is $\frac{300}{v-10}$.

$\frac{300}{v-10} - \frac{300}{v} = 1$ $\frac{300v - 300(v-10)}{v(v-10)} = 1$

$3000 = v^2 - 10v$ $v^2 - 10v - 3000 = 0$ $(v-60)(v+50) = 0$

$v = 60$ km h^{-1}

11. Let speed of the boat relative to the water is v
and speed of the water is u.

$\frac{2}{v+u} = 8$ $\frac{2}{v-u} = 12$ $v+u = \frac{1}{4}$ $2v = \frac{1}{4} + \frac{1}{6}$

                                  $v-u = \frac{1}{6}$ $2u = \frac{1}{4} - \frac{1}{6}$

$v = \frac{5}{24} = 0.21$ km min^{-1} $u = \frac{1}{24} = 0.04$ km min^{-1}

12. Equation of the motion of the slower car is $x = 60t$, equation of the motion of the faster car is $x = 195 - 70t$.

When they met $\quad 60t = 195 - 70t \qquad 130t = 195$

$t = 1.5\,h$

$x = 60 \times 1.5 = 90\,km$

13. Let distance one way is S. Time upstream $\dfrac{S}{8}$, time downstream is $\dfrac{S}{12}$. Total time $\dfrac{S}{8} + \dfrac{S}{12} = 5$

$\dfrac{3S + 2S}{24} = 5 \qquad 5S = 120 \quad S = 24\,km$

14. Let speed of the stream is u.

$\dfrac{6}{12+u} - \dfrac{6}{12+2u} = \dfrac{4}{60} \qquad \dfrac{6(12+2u) - 6(12+u)}{(12+u)(12+2u)} = \dfrac{4}{15}$

$\dfrac{6u}{(12+u)(12+2u)} = \dfrac{1}{15} \qquad \overset{90}{\cancel{90}u} = 144 + 36u + 2u^2$

$u^2 - \overset{27}{\cancel{27}}u + 72 = 0$

$(u - 3)(u - 24) = 0$

$u = 3\,km\,h^{-1} \qquad u = 24 - \text{rejected as } 24 > 12$

15. Let boy's speed in still water is v

$\dfrac{8}{v-3} + \dfrac{8}{v+3} = 5 \qquad 8(v+3) + 8(v-3) = 5(v^2 - 9)$

$16v = 5v^2 - 45$

$5v^2 - 16v - 45 = 0$

$5v^2 - 25v + 9v - 45 = 0$

$5v(v-5) + 9(v-5) = 0$

$(v-5)(5v+9) = 0$

$v = 5\,km\,h^{-1}$

16.

$$\frac{125}{V-10} - \frac{125}{V} = \frac{25}{60}$$

$$\frac{5}{V-10} - \frac{5}{V} = \frac{1}{60}$$

$$\frac{5V - 5(V-10)}{V(V-10)} = \frac{1}{60}$$

$$3000 = V^2 - 10V$$

$$V^2 - 10V - 3000 = 0$$

$$(V-60)(V+50) = 0$$

$$V = 60 \ km \ h^{-1}$$

17. Let father's speed is V. Daughter's speed is $V+6$.

Time is the same.

$$\frac{12}{V} = \frac{14}{V+6}$$

$$12(V+6) = 14V$$

$$2V = 72$$

$$V = 36 \ km \ h^{-1}$$

18.

$$\frac{S}{105} - \frac{S}{120} = \frac{1}{3}$$

$$\frac{8S - 7S}{840} = \frac{1}{3}$$

$$S = \frac{840}{3} = 280 \ km$$

19. Let speed of the slower hiker is V. Faster hiker speed is $V+1$.

$$\frac{15}{V+1} - \frac{10}{V} = \frac{1}{2}$$

$$\frac{15V - 10(V+1)}{V(V+1)} \quad \frac{1}{2}$$

$$\frac{5V - 10}{V^2 + V} = \frac{1}{2}$$

$$10V - 20 = V^2 + V$$

$$V^2 - 9V + 20 = 0$$

$$(V-4)(V-5) = 0$$

$$V = 4 \ km \ h^{-1} \quad or \quad V = 5 \ km \ h^{-1}$$

20. Equation of the motion of the man is $x = 4t$

Equation of the motion of the son is $x = 10\left(t - \frac{1}{4}\right)$

$$4t = 10\left(t - \frac{1}{4}\right)$$

$$6t = \frac{5}{2}$$

$$t = \frac{5}{12} \ h = 25 \ min$$

Answer: at 8:25