Momentum

Study Design

- **investigate and apply theoretically and practically the laws of energy and momentum conservation in isolated systems in one dimension.**

Introduction

The momentum (**p**) of a body is the product of its mass and velocity.

$$\mathbf{p} = m\,\mathbf{v}.$$

The unit is kilogram metre per second (kg m s^{-1}) or Newton seconds ($N\ m^{-1}$)
Momentum is a vector. It has a magnitude and a direction.
Momentum is conserved when no external force acts. It is transferred to the earth whenever a body hits the ground or slides to a halt.

Impulse

Deductions from Newtons second law.
Consider a body of mass 'm' changing its velocity from 'u' to 'v' in time 't' under the action of a constant force F.
From Newton's second law of motion,

$$\mathbf{F} = m\,\mathbf{a}, \text{ since } \mathbf{a} = \frac{\mathbf{v} - \mathbf{u}}{\Delta t} \qquad \mathbf{F} = \frac{m\mathbf{v} - m\mathbf{u}}{\Delta t}$$

$$\therefore\ \mathbf{F}\,\Delta t = m\,\mathbf{v} - m\,\mathbf{u}$$

The product of a constant force and the time for which it acts is called the **impulse** (**I**) of the force.

$\mathbf{I} = \mathbf{F}\,\Delta t$ The unit is the Newton-second. (N s)

Impulse is the change in momentum i.e. $\mathbf{I} = \mathbf{p_2} - \mathbf{p_1}$.

Thus the impulse can be measured by the change in momentum produced. **Impulse** and **momentum** are vectors. So whenever a force acts, the direction of all the following is the same:

$\mathbf{F}$, $\mathbf{a}$, $\Delta\mathbf{v}$, $\mathbf{F}.\Delta t$, $\Delta\mathbf{p}$.

Notes on problem solving
1. As momentum is a vector, a sign convention in problems is essential.
2. The negative sign for the change in momentum indicates a loss of momentum.

Remember that **F** is the resultant forward force.

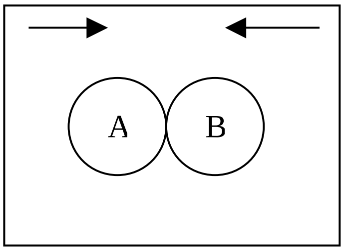
Notes
1. Calculations by either formula or graph involve **v** and $\Delta\mathbf{v}$. In many cases the body starts from rest and then, and only then, does $\Delta\mathbf{v}$ equal the actual velocity, **v.**
2. If asked for "**p**" look for impulse, if asked for impulse look for "**p**".
3. If answering for impulse, the units are "N s" if answering for "**p**", the units are "kg m s^{-1}".

<u>Graphically - constant or non-constant forces.</u>
Since $\mathbf{F}.\Delta t = m\ \Delta \mathbf{v}$
for a constant force, it follows that the impulse will always be given by the area under the force-time graph. This area also measures the change in momentum.

Area under "**F - t**" graph = **Impulse** = Δ **momentum.**
The slope of a momentum-time graph gives the force, and the area under a force-time graph equals the change in momentum.

Conservation of momentum

When A and B collide, the action on A by B is equal and opposite to that on B by A. (Newtons 3rd)



Hence the rate of change of momentum of A is equal and opposite to the rate of change of momentum of B. Since the time of contact is the same for both, then the change in momentum of A is equal and opposite to the change in momentum of B.

That is, THE TOTAL MOMENTUM BEFORE IMPACT EQUALS THE TOTAL MOMENTUM AFTER IMPACT.

This is known as the law of conservation of momentum. $\mathbf{P}_{(total)}$ is constant before, during and after the collision.

<u>Notes.</u>

1. Remember that a sign convention is essential.
2. If the bodies collide and stay together, then the momentum after the collision $\mathbf{p}_{final} = \Sigma\ \mathbf{p}_{\mathbf{initial}}$
   $\Sigma m\ \mathbf{v}_{final} = \Sigma m\ \mathbf{v}_{initial}$
3. Mathematically, problems on 'collision' or 'explosion' are similar, except that for an explosion, the momentum of the system before the blast is often zero.
4. $\mathbf{p}_{(total\ before\ the\ collision)} = \mathbf{p}_{(total\ after\ the\ collision)}$
5. Always draw a diagram
6. Any unit may be used for mass or velocity, as long as such units are consistent within the equation.

<u>Momentum transfer involving the Earth</u>

1. Body rises under gravity - slows down and loses momentum to the earth.
2. Body falling under gravity - speeds up giving the earth equal and opposite momentum change.
3. Falling body hits the ground - its **p** is transferred to the earth.
4. Body slowed due to friction - gives the earth and equal and opposite **p**.
5. Body accelerated due to friction - gives the earth an equal and opposite **p**.

Momentum & Impulse

How does a karate expert chop through cement blocks with a bare hand?
Why does a fall onto a wooden floor hurt less than onto a cement floor?
Why do people in larger vehicles usually end up with fewer injuries in accidents?

It's easy to come up with answers like…

"The karate guy is strong!"
"Wood is softer!"
"Bigger is better!"

but have you ever stopped to consider the why? That's when physics comes walking in, waving explanations in everyone's face.

Spend a couple minutes right now to come up with explanations of the three situations using physics principles you have learned so far. Keep these situations and your explanations in mind as you cover this section on momentum and impulse. See if you need to modify or change your explanations based on what you learn.

Momentum Impulse Newton Graphing

Momentum

Momentum is an idea that combines mass and velocity into one package. It is an idea that is similar to **inertia** and **kinetic energy**.

> **Inertia** is the property of an object to stay at rest or in motion.
>
> **Kinetic energy** is the amount of energy that an object due to its motion. ($E_k = ½ mv^2$)

Momentum is not truly either of these, but ends up like a mix of the two.

- If you compare and contrast momentum and **kinetic energy**, you'll notice a couple things…
  - First, they both have mass and velocity in their formulas.
  - Second, **kinetic energy** has to do with ability to do work, momentum doesn't.
  - Although they are similar, they are not the same.
- We haven't given you any way to calculate inertia yet, so is momentum the same as inertia?
  - Not really. **Inertia** is a concept, not something that is directly measured.

Momentum is calculated by multiplying the mass and velocity of an object.

$$p = m\,v$$

p = momentum (kg m/s)
m = mass (kg)
v = velocity (m/s)

Notice that momentum does not have a nice derived unit, although I would appreciate it if you lobbied physicists to name it the "Clintberg" in my honor. For now you'll just need to give the units "kg m/s"

Example 1: A 1000 kg car is moving at 10km/h. **Determine** the momentum of the car.

p = mv
p = 1000kg (2.78m/s)
p = 2.78e3 kg m/s

Momentum Impulse Newton Graphing

Impulse

The simple definition for impulse is that it is a change in momentum.

- If an object's velocity changes, there is a change in momentum, so there must be an impulse
- We assume that you are not going to change the mass of an object.

$$\Delta p = m\ \Delta v$$

Δp = impulse (change in momentum) (kgm/s)
m = mass (kg)
Δv = change in velocity ($v_f - v_i$) (m/s)

Example 2: A box of tic tacs (15g) is sliding along the table at 5.0m/s. I try to stop it, but only slow it down to 1.6 m/s. **Determine** the impulse I impart to the box.

$\Delta p = m\ \Delta v$

$\Delta p = m\ (v_f - v_i)$

= 0.015kg (1.6m/s – 5.0m/s)

Δp = -0.051 kg m/s

The negative sign just identifies that my impulse was in the negative direction. Momentum was **taken away** from the object.

But wait a second, if an impulse changes the velocity of the object, that means it's ***accelerating***.

- Acceleration of an object can only occur if a force is acting on the object… so force must be related to impulse in some way.
- This leads us to the link to Newton.

Momentum Impulse Newton Graphing

Newton

When Newton came up with his 2nd Law of motion, he didn't write it in the form we usually see it today, F = ma.

- Remember that he was playing around with some new ideas, and didn't necessarily look for the "easiest" way to state his theories.
- Instead, he kept talking about the "quantity of motion" of an object, what we today call momentum.
- When he stated his 2nd law he said the force is proportional to the rate of change in the momentum.

$$F = \frac{\Delta p}{\Delta t}$$

Notice that you can solve this formula to get what we now consider the "standard" form of the 2nd law…

$F = \frac{\Delta p}{\Delta t}$ but we know that $\Delta p = m\,\Delta v$

$F = \frac{m\,\Delta v}{\Delta t}$ and we also know that $a = \frac{\Delta v}{\Delta t}$

$$F = ma$$

We can also come up with a different (and more versatile) version of the impulse formula.

- The formula you were first given was…

$$\Delta p = m\,\Delta v$$

- But we just saw that Newton used impulse in his formulas…

$$F = \frac{\Delta p}{\Delta t}$$

which becomes

$\Delta p = F\,\Delta t$ (which can be a useful formula!)

We can stick these two formulas together to get

$$F\,\Delta t = m\,\Delta v$$

You can see that a change in momentum (impulse) depends on two factors… force and time interval.

- To change an object's momentum, think of the following situations:

    1. You could apply a medium force over a medium time interval.

$$F \, \Delta t = \Delta p$$

    2. You could apply a big force over a small time interval and get the same impulse as in (1).

$$\Huge F \, {\scriptstyle \Delta t} \normalsize = \Delta p$$

    3. Or, you could apply a small force over a long time interval and still get the same impulse.

$${\scriptstyle F} \, \Huge \Delta t \normalsize = \Delta p$$

This explains why you would want to come to a stop by hitting a haystack instead of a brick wall with your car.

- In each case the impulse is the same (your mass stays the same, your Δv stays the same).
- When you hit the brick wall…

$$\Huge F \, {\scriptstyle \Delta t} \normalsize = \Delta p$$

- Youch! All that force on your body is going to hurt! The impulse happened in a very short time period.
- When you hit the haystack…

$${\scriptstyle F} \, \Huge \Delta t \normalsize = \Delta p$$

- Not much force at all, since the impulse is spread out over a long time period!

It's the force that "hurts", so you want it to be as small as possible.

- You can use the same argument to explain hitting an airbag instead of a steering wheel, using a bungee cord instead of a rope, or falling onto a wooden floor instead of a cement one.

Example 3: A 75kg man is involved in a car accident. He was initially traveling at 65km/h when he hit a large truck.

a) If he had no airbag in his car and he came to rest against the steering wheel in 0.05s, **determine** how much force was exerted on his body.

First, change 65 km/h into 18m/s.

$F \Delta t = m \Delta v$
$F = (m \Delta v) / \Delta t$
$= (75kg)(-18m/s) / (0.05s)$
$F = -2.7e4$ N

$\Delta v = v_f - v_i = 0 - 18 = -18m/s$

b) If he did have an airbag that inflated and deflated correctly, bringing him to rest over a time of 0.78s, **determine** how much force was exerted on his body.

$F \Delta t = m \Delta v$
$F = (m \Delta v) / \Delta t$
$= (75kg)(-18m/s) / (0.78s)$
$F = -1.7e3$ N

Which is only about 6% of the force felt without an airbag… a definite improvement!

Momentum Impulse Newton Graphing

Graphing

At times it can be useful to graph Force vs. Time to determine impulse.

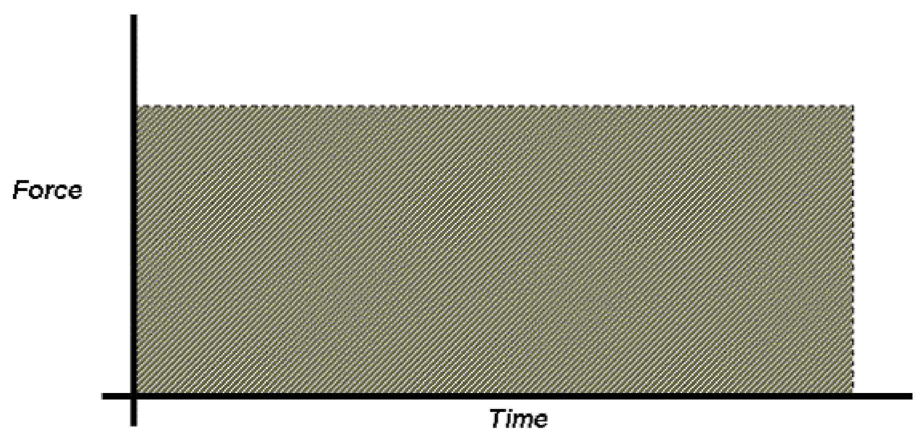


Illustration 1: Graph of Force as a function of Time

What is the area under the graph?

Area = base X height
$= F \Delta t$
Area = Impulse

Example 4: I am in a car that is accelerating. I want to calculate the impulse that is acting on the car during this time of 5.78s. If I know that the force on the car increases from 0 N to 3012 N over this time, calculate the impulse.

Let's start by graphing the information we were given…

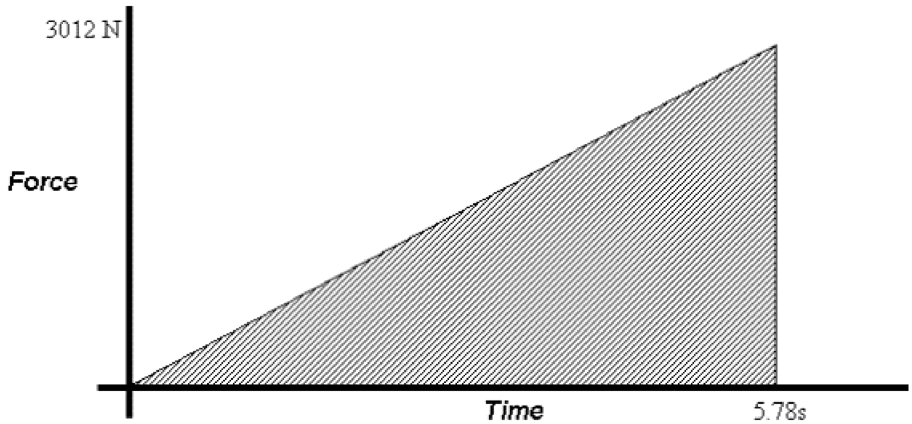


Illustration 2: Graph for Example 2 (Force as a function of Time)

If we calculate the area under the graph (in this case a triangle) we will know what the impulse is.

A = ½ bh
= ½ (5.78 s)(3012 N)
A = 8.70e3 kgm/s

Even if it is a curved line, you can still at least estimate the area under the graph…

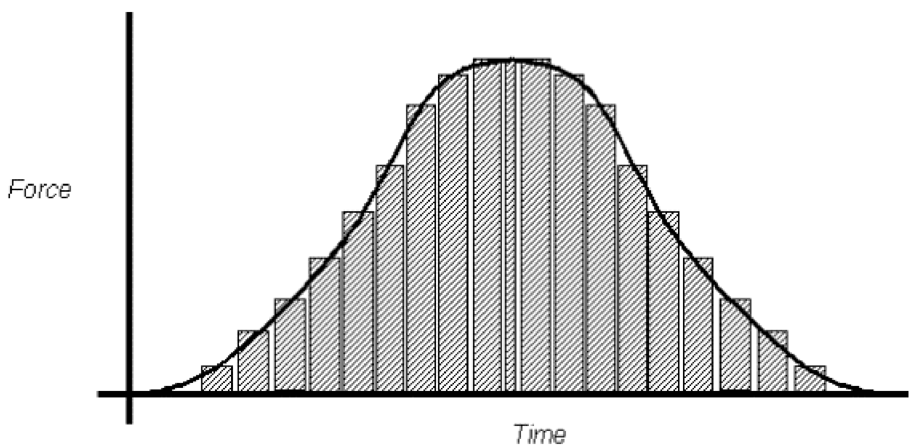


Illustration 3: Curved Line Graph of Force as a function of Time

- For a curved line like this one, we can figure out the area of a bunch of little columns under the line to approximate the true area.
- If you've taken a calculus course then you'll know that you can make better approximations by assuming an infinite number of columns under the line.
- You are not responsible for calculus calculations in this course… we'd just want you to approximate the values as closely as possible.