Length contraction. Solutions

VCAA 2005

A container inside a rocket ship is observed through a window by Sam, an astronaut, **floating freely in space**. Sam observes the rocket ship travelling past at a constant speed VR = 0.2 c. The dimensions of the container, **as measured by astronauts inside the rocket ship**, are shown in Figure 1, and are

- the proper length L_0 (parallel to the direction of motion of the rocket ship)
- the proper width W_0 (perpendicular to the direction of motion of the rocket ship)
- the proper height H_0 (perpendicular to the direction of motion of the rocket ship).

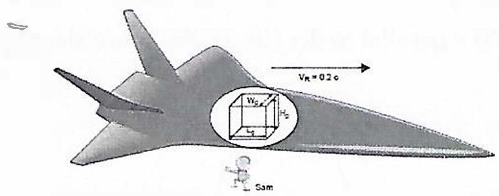

Figure 1

√

Question 3
Which of the options **(A.D)** best describes its dimensions **as observed by Sam**?

A. $L < L_0$, $W < W_0$, $H < H_0$

B. $L < L_0$, $W = W_0$, $H = H_0$

C. $L > L_0$, $W = W_0$, $H = H_0$

D. $L < L_0$, $W < W_0$, $H = H_0$

VCAA 2007

Ann and Bill are travelling in space in identical spaceships as shown in Figure 1 below. Ann knows that each spaceship is of proper length L_0. At some time Ann observes Bill's ship passing her with a speed v, and she measures its length

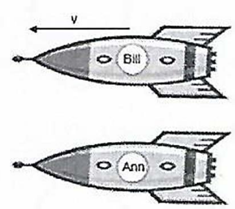

Figure 1

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Question 5
Which of the following **(A–D)** best represents the length (L) of Bill's ship as measured by Ann?

A. $L = \dfrac{L_0}{\sqrt{1 - v^2/c^2}}$

B. $L = L_0\sqrt{1 - v^2/c^2}$

C. $L = \dfrac{L_0}{\sqrt{1 + v^2/c^2}}$

D. $L = L_0\sqrt{1 + v^2/c^2}$

Ann, who is studying VCE physics, thinks about this. If she measures Bill's ship larger than L_0, then he must measure hers as smaller than L_0. Conversely, if Ann measures Bill's ship smaller than L_0, then Bill must measure Ann's as larger than L_0.

Question 6

Is Ann's reasoning correct? Write your answer in the box provided, and in the space below explain the logic of your answer

Not correct. L always smaller than Lo.

VCAA 2008

Use the following information to answer Questions 1 and 2.

A rocket passes a space station at a speed 0.80 c parallel to the side of the space station. There is a circular window on the space station, as shown in Figure 1.

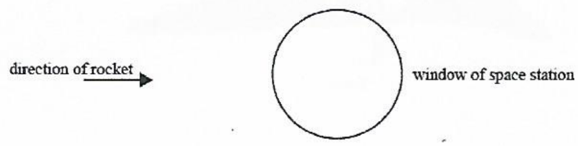


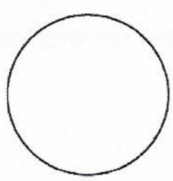
Figure 1

A person inside the passing rocket observes the window.

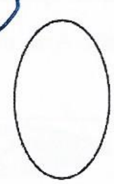
Question 1

Which of the following figures best shows how the window would look to the person on the passing rocket?

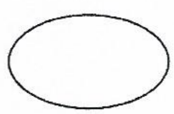
A.



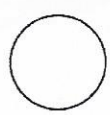
B.



C.



D.



Question 2

For which of the following values of the Lorentz factor, γ, would relativistic changes in time, length and mass **not** be observed?

A. close to 0

B. significantly less than 1

C. approximately equal to 1

D. significantly greater than 1

When $v = 0$ $\frac{v}{c} = 0$

$1 - \frac{v^2}{c^2} = 1$ $\gamma = 1$

$t = t_0$ $L = L_0$

Figure 2 below shows Fred in a futuristic train travelling at constant relativistic speed in an easterly direction in a straight line. Fred is halfway between two people, Alan and Bob, who are at opposite ends of the carriage.

The train passes a platform. Nancy is standing on the platform.

At the instant that Fred and Nancy are directly opposite each other, Fred sees both Alan and Bob strike matches simultaneously.

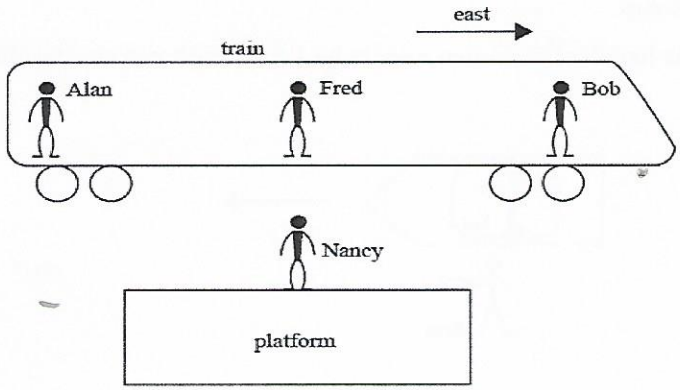


Figure 2

Question 4

Fred measures the carriage he is travelling in to be 20 m long. Nancy has measured the platform she is standing on to be 10 m long. The train rushes past at such a speed that Nancy sees the carriage and the platform to be the same length.

How fast was the train moving?

A. 0.50 c　　**B.** 0.75 c　　**C.** 0.87 c　　**D.** 0.97 c

$\gamma = \frac{20}{10} = 2$　　$2 = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$　　$\frac{1}{4} = 1 - \left(\frac{v}{c}\right)^2$　$\left(\frac{v}{c}\right)^2 = \frac{3}{4}$　　$v = 0.87c$

Question 9

Roger is in a spacecraft travelling at constant speed, passing Bridget who is stationary. Roger is holding a ruler. Roger measures its length to be 1.0 m. The ruler can be seen by Bridget through a window as the spacecraft passes. Bridget measures the length of the ruler to be 0.8 m.

Which one of the following gives the proper length of the ruler?

A. 0.20 m　　**B.** 0.80 m　　**C.** 1.0 m　　**D.** 1.2 m

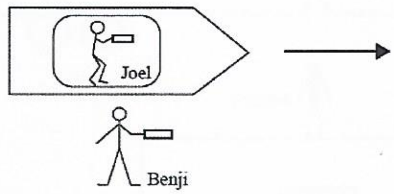
Proper length measured by observer stationary to the object

VCAA 2009

The following information relates to Questions 2–4.

Astronaut Benji is floating freely in space when Joel's spaceship passes him at high speed. Benji is holding a ruler, and can see through the window of the spaceship that Joel is also holding a ruler. The two rulers are parallel to the direction of motion of the spaceship.

Both Benji and Joel measure the length of their own ruler to be 1.0 m. Each can see the other ruler. Benji observes Joel's ruler to be 0.6 m long.



Question 2

Which one of the following gives the length of Benji's ruler as observed by Joel?

A. 0.6 m **B.** 1.0 m **C.** 1.4 m **D.** 1.7 m

As their speed relative to each other is the same, length contraction will be the same

Question 3

Which one of the following gives the proper length of Joel's ruler?

A. 0.6 m **B.** 1.0 m **C.** 1.4 m **D.** 1.6 m

Proper length of Joel's ruler measured by Joel

Question 4

Which one of the following gives the speed of Joel's spaceship as measured by Benji?

A. 0.4 *c* **B.** 0.6 *c* **C.** 0.8 *c* **D.** *c*

$\gamma = \frac{1}{0.6}$ $L = L_0 \times 0.6$ $\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{0.6}$ $1-\left(\frac{v}{c}\right)^2 = 0.6^2$ $\frac{v}{c} = 0.8$

Question 5

According to Einstein's theory of special relativity, which one of the following does not depend on the motion of the observer?

A. the order of occurrence of two events

B. the time interval between two events

C. the distance between two points

D. the speed of light

VCAA 2010

On a planet a long way away, a racing car is moving at high speed ($0.9c$) along a straight track. It is heading straight for a post. Jim is standing next to the post. The situation is shown in Figure 1. Racing car is 1 km from the post

Figure 1

Question 2

The driver of the racing car, Susanna, measures the distance between herself and the post at exactly the same time that she sends the flash of light.

Which one of the following is closest to the distance that she measures?

A. 0.44 km **B.** 0.90 km **C.** 1.00 km **D.** 2.29 km

$L = \frac{L_0}{\gamma}$ $L = 1000 \times \sqrt{1 - 0.9^2} = 0.44\,\text{km}$

Question 4

Which one of the following is the best description of the **proper length** of an object travelling with constant velocity?

A. The length when measured by any observer at the same location.

B. The length when measured by an observer at rest relative to the object.

C. The length when both ends of the object are measured at the same time.

D. The length when measured with a proper standard measuring stick.

Question 5

Two physics students are conducting accurate experiments to test Newton's second law of motion ($\Sigma F = ma$). Each student is in a windowless railway carriage. One carriage (carriage A) is moving at a constant velocity of $0.9c$. The other carriage (carriage B) is moving at 10 m s–1 and decelerating.

Which one of the following best describes the likely results of their experiments?

A. Only the experiment in carriage A confirms Newton's second law of motion.

B. Only the experiment in carriage B confirms Newton's second law of motion.

C. Neither experiment confirms Newton's second law of motion.

D. Both experiments confirm Newton's second law of motion.

Frame B accelerates, so laws of physics are different.

VCAA 2011

Question 4

A tower is 300 m tall. A cosmic fray particle is travelling from the upper atmosphere. It is travelling perpendicular to the ground. As it speed past the top of the tower, it measures the height of the tower to be 200m.
Which of the following potions is the best estimate of the speed of the cosmic ray?

A. 0.56c **B.** 0.67c **C.** **0.75c** **D.** 0.81

$L = \frac{L_o}{\gamma}$ $\gamma = \frac{300}{200} = 1.5$ $1.5 = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ $1-\left(\frac{v}{c}\right)^2 = \frac{4}{9}$

$v = 0.75c$

Question 11

Which of the following statements about **proper length** is most accurate?

A. The proper length of an object is always greater than or equal to another measure of the length of the object. A

B. The proper length of an object is always less than another measure of the length of the object.

C. The proper length of an object is sometimes less than another measure of the length of the object and sometimes greater than or equal to another measure of the length of the object.

D. The proper length of an object can only be measured by an observer who is moving relative to the object.

VCAA 2013

Question 5

A physicist purchased a limousine, but found that it was twice as long as her garage. She reasoned that in the garage's reference frame, it should be possible for a moving limousine to fit exactly inside the garage for an instant. What is the minimum speed at which the limousine would have to travel in order for this to work?

A. $\frac{c}{\sqrt{2}}$

B. $\frac{3}{4}c$

C. $\frac{\sqrt{3}}{2}c$

D. $\sqrt{3}c$

$L = \frac{L_o}{\gamma}$ $\gamma = 2$

$2 = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ $1-\left(\frac{v}{c}\right)^2 = \frac{1}{4}$

$v = \frac{\sqrt{3}}{2}c$

Question 9

Lucy is on a train travelling at 0.8c. The train passes Edmund, who is standing on the platform at a train station. They each measure a different length for the train and also measure a different length for the platform.

Which one of the following statements is correct?

A. Lucy measures a proper length for the train because she is stationary with respect to the train.

B. Edmund measures a proper length for the train because he is stationary in his reference frame.

C. Edmund measures a proper length for both the platform and the train because he is standing still.

D. Lucy measures a proper length for the platform because she passes the start and the end of the platform over the course of her journey.

VCAA 2014

Question 3

A person moving parallel to the length of a 2.00 m ruler observes the change of length of this ruler due to relativity to be 0.010 m.

What is the person's speed relative to the ruler?

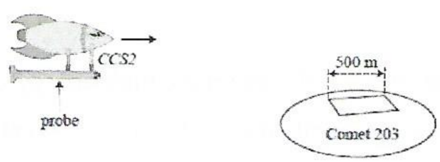
A. 0.0050c **B.** 0.010c **C.** 0.10c **D.** 0.90c

$L = 2 - 0.01 = 1.99$ $\quad \frac{2}{1.99} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ $\quad v = 0.1c$

VCAA 2015

Use the following information to answer Questions 7 and 8.

Comet-chasing spacecraft *CCS2* travels at a speed for which $\gamma = 1.5$ relative to the nearest stars. It approaches Comet 203, which is effectively stationary relative to the nearest stars, as shown in Figure 2. There is a landing probe attached to *CCS2*.



Question 7

The designated landing area on the comet has length 500 m in the comet's frame and is parallel to spacecraft *CCS2*'s velocity.

What is the length of this landing area, as measured by instruments on *CCS2*?

A. 750 m

B. 500 m

C. 408 m

D. 333 m

$L = \frac{L_0}{\gamma} = \frac{500}{1.5} = 333\ m$

Question 8 (82%)
Spacecraft *CCS2* releases a probe that will land on the comet. Near touchdown, the probe is at the same velocity as the comet.

Which one of the following is able to measure the proper length for the landing area?

A. the probe when it is travelling at the same velocity as the comet

B. *CCS2* because it has far more accurate radar instruments than the probe

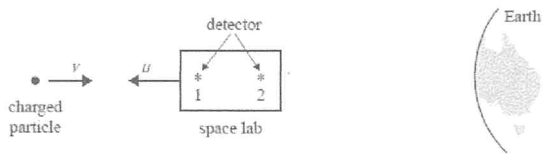
C. *CCS2* at the instant it passes by the landing area on the comet

D. a radar pulse from *CCS2* because the pulse will momentarily be stationary when it bounces off the landing area.

Proper length is measured by observer stationary to what is measured

VCAA 2016

Question 7 (58%)



A space lab travelling at $u = 0.8c$ ($\gamma = 1.67$) away from Earth can record high-energy charged particles passing through its detectors. One particle is travelling towards Earth at $v = 0.91c$ ($\gamma = 2.4$) **relative to the space lab**.

Two detectors, numbered 1 and 2 in Figure 4, are 2.0 m apart in the space lab's frame.

How far apart are the two detectors in this particular particle's frame?

A. 0.83 m **B.** 1.2 m **C.** 3.3 m **D.** 4.8 m

$$L = \frac{L_0}{\gamma} = \frac{2}{2.4} = 0.83$$

VCAA 2017
Question 10 (2 marks) **53%**
The length of a spaceship is measured to be exactly one-third of its rest length as it passes by an observing station.
What is the speed of this spaceship, as determined by the observing station, expressed as a multiple of c?

$$L = \frac{L_0}{\gamma} \qquad \gamma = \frac{L_0}{L} = 3$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad 1-\left(\frac{v}{c}\right)^2 = \frac{1}{9} \qquad \left(\frac{v}{c}\right)^2 = \frac{8}{9} \qquad v = 0.94c$$

Question 11

Tests of relativistic time dilation have been made by observing the decay of short-lived particles. A muon, travelling from the edge of the atmosphere to the surface of Earth, is an example of such a particle.

To model this in the laboratory, another elementary particle with a shorter half-life is produced in a particle accelerator. It is travelling at $0.99875c$ ($\gamma = 20$). Scientists observe that this particle travels 9.14×10^{-5} m in a straight line from the point where it is made to the point where it decays into other particles. It is not accelerating.

b. (2 marks) **38%**

Calculate the distance that the particle travels in the laboratory, as measured in the particle's frame of reference.

$$L = \frac{L_o}{\gamma} = \frac{9.14 \times 10^{-5}}{20} = 4.6 \times 10^{-6} \text{ m}$$

c. (3 marks) **8%**

Explain why the scientists would observe more particles at the end of the laboratory measuring range than classical physics would expect.

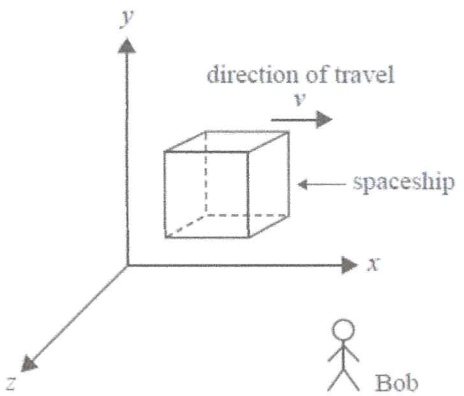
Due to the particle's velocity distance to the detector in particle's frame of reference is reduced. Therefore more particles will be able to reach detector before they decay.

VCAA 2019 NHT

Question 18 (3 marks)

Alien astronauts are travelling between star systems aboard a cube-shaped spaceship, as shown in Figure 16. The sides of the cube along the *x*-axis, *y*-axis and *z*-axis measure 3.20×10^3 m in the spaceship's frame of reference.

The spaceship passes Bob, who is on a space station, at speed $v = 0.990c$ ($\gamma = 7.09$).



$$L = \frac{L_o}{\gamma} = \frac{3.2 \times 10^3}{7.09} = 4.5 \times 10^2 \text{ m}$$

Figure 16

In the table below, determine the dimensions of the cube-shaped spaceship as measured from Bob's frame of reference and explain your reasoning.

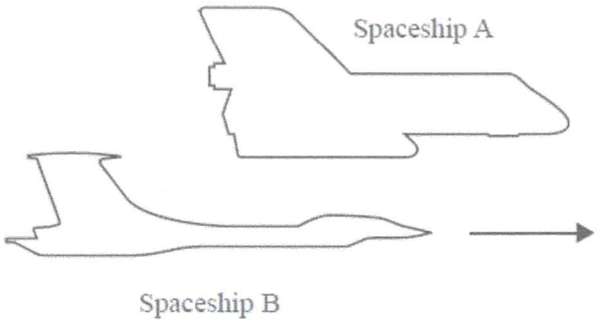
length of side along *x*-axis	4.5×10^2	m
length of side along *y*-axis	3.2×10^3	m
length of side along *z*-axis	3.2×10^3	m

Reasoning Length contraction only occurs in the direction of travel, so only x-component is reduced

VCAA 2019

Question 13 M/C

Joanna is an observer in Spaceship A, watching Spaceship B fly past at a relative speed of $0.943c$ ($\gamma = 3.00$). She measures the length of Spaceship B from her frame of reference to be 150 m.



Which one of the following is closest to the proper length of Spaceship B?

A. 50 m

B. 150 m

C. 450 m

D. 900 m

$L_0 = L\gamma = 150 \times 3$