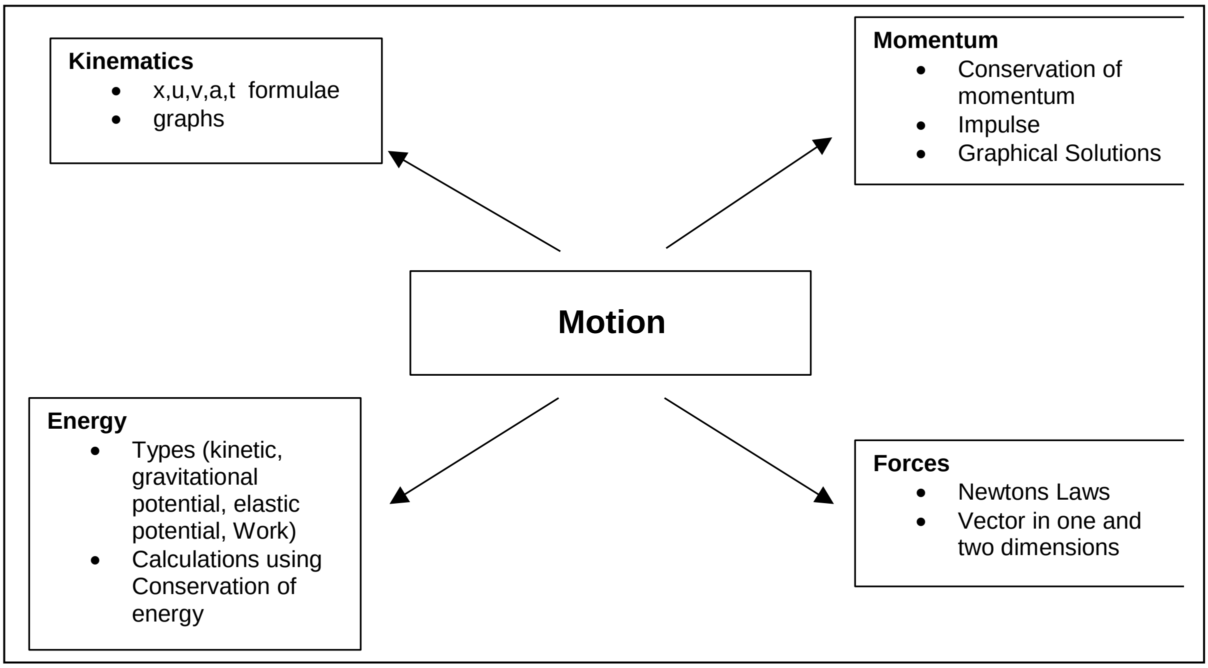
Kinematics

Introduction



When solving problems, you will have to ***choose*** which concept of physics (Kinematics, Momentum, Energy or Forces) will be most useful in solving the problems.

Some problems you will be only able to solve using one concept, others can be solved in more than one way, but may be very simple using a particular technique. Hence you must be able to solve problems using all of the concepts, and you must develop an instinct for choosing the most efficient path.

You will also need to decide whether to use a graphical or numerical technique to solve problems.

Definitions

From your prior studies of motion you should be familiar with the following kinematic definitions:

Distance Travelled - How far an object has moved in total during its motion. (m).
Displacement – How far an object is at from a reference position. (m)
Speed – How fast an object is moving. (m/s)
Velocity – How fast an object and what direction an object is moving in. (m/s)

Acceleration – The rate at which the velocity of an object is changing i.e. how many (m/s) the velocity of an object is changing by every second. Acceleration also has a direction

These physical quantities can be divided into two categories, scalars and vectors.

Vectors: Vectors are quantities that have a magnitude and a direction.
E.g. displacement, velocity, and acceleration.

Scalars: Scalars are quantities that only have a magnitude,
E.g. speed and distance travelled.

Average Quantities

You will occasionally be asked to determine average quantities. For example, you may be asked to determine the velocity, on average, at which a car was moving between two times. Average velocity and acceleration are determined using the following formulae.

$$\text{average velocity} = \frac{\text{total displacement}}{\text{time taken}} \qquad v_{av} = \frac{x_2 - x_1}{\Delta t}$$

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}} \qquad a_{av} = \frac{v - u}{\Delta t}$$

where

x_2 is the final position, **x_1** is the initial position, **Δt** is the time period,
v is the final velocity and **u** is the initial velocity.

Graphs

You need to be able to use a wide range of graphs. When given a graph in the exam, look for the following on the graph before even reading the question:

- type of graph (F – d, F – v, Energy – distance, F – t etc.).
- the units on the axis.
- the limit reading on each axis.
- Look at the scale on both axes, be aware for anything non-standard
- Think about what is given by a direct reading from the graph, the gradient of the graph and the area under the graph

In Year 11 Physics it is typical to restrict the types of graphs that you experience to those with ‘time’ usually on the horizontal axis. Expect to find ‘distance’ and others on the horizontal axis in Year 12.

Graphical Techniques

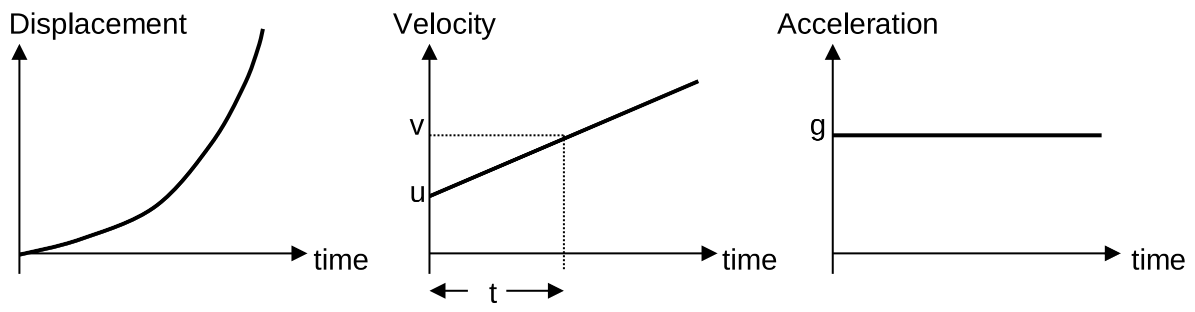
In kinematics you can be asked to interpret several graphs. Graphs can be used to determine *instantaneous* quantities i.e. the value of a quantity at a specific time. For example, a velocity time graph (v-t) can be used to determine how fast an object was moving at a specific time. It could also be used to determine how far the object has moved up to that time (by finding the area under the curve) or its acceleration (by determining the gradient at a specific point). The type of information that can be determined from different graphs is summarised in the following table.

Graph type → / Found from	x - t	v - t	a – t
Direct reading	'x' at any 't' 't' at any 'x'	'v' at any 't' 't' at any 'v'	'a' at any 't' 't' at any 'a'
Gradient	Instantaneous velocity at any point. V_{av} between any two points	Instantaneous 'a' Average 'a'	Meaningless
Area under graph	Meaningless	Δx	Δv

The gradient at a particular time is determined by drawing a tangent line to the curve at that point, and then determining the gradient of the tangent line.

Constant Acceleration

Consider the following series of graphs. These illustrate the relationships mentioned in the table above. Notice that the velocity – time graph is the gradient of the displacement – time graph, and the acceleration – time graph is the gradient of the velocity – time graph.



Scalar and vector quantities

A scalar quantity requires a numerical value and a unit to specify it, e.g. distance 6.5 km and mass 10 kg are scalar quantities.

Example 1 Name two more scalar quantities.

Surface area, e.g. 12 m^2; air pressure, e.g. 0.5 kPa

A vector quantity requires a numerical value together with a unit and a direction to specify it completely. The numerical value with the unit is called the **magnitude** of the vector quantity. Examples of vector quantities are: force, 9.8 N left; velocity, 70 km h^{-1} N35°W.

Example 2 Name two other examples of vector quantities.

Gravitational field, e.g. 9.8 N kg^{-1} downward; momentum change, e.g. 2.0 kg ms^{-1} SE

Vector quantities in one dimension

In one dimension, a positive or negative sign is used to indicate the direction of a vector quantity. Usually positive is chosen for to the right or upward direction, e.g. a force of 5 N to the left is written as ${}^-5$ N; an upward velocity of 20 ms^{-1} is written as ${}^+20$ ms^{-1}.

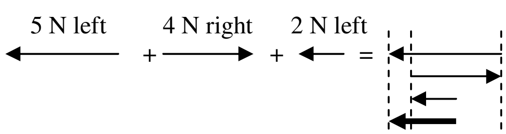
Vector quantities can also be represented by arrows drawn to scales. The length of the arrow shows the magnitude, and the direction is shown by the arrow head.

Addition of vector quantities in one dimension

Example 1 Three forces act on the same object: 5 N left, 4 N right and 2 N left. Find the **net force** on the object.

Consider vector quantities as **directed numbers**:
${}^-5 + {}^+4 + {}^-2 = {}^-3$ N, i.e. 3 N left.

Graphically: When vector quantities are represented by arrows, addition is done by placing the head of the second arrow to the tail of the first arrow. This is repeated if more than two arrows are involved. The **resultant** is an arrow starting from the tail of the last arrow to the head of the first.



Resultant (net force) is 3 N left.

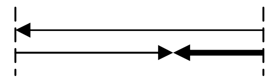
Note: The order that this is carried out does not affect the resultant.

Example 2 A car travels to the west for 1.2 km and then to the east for 0.7 km. Find the position of the car from its starting point.

As directed numbers: Take east as the positive direction.

${}^-1.2 + {}^+0.7 = {}^-0.5$, i.e. 0.5 km west of the starting point.

Graphically:



Subtraction of vector quantities in one dimension

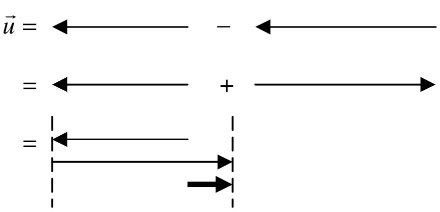
Example 3 The velocity of a car is reduced from 75 km h^{-1} west to 60 km h^{-1} west. What is the change in velocity of the car?

As directed numbers: Take east as the positive direction.

$\Delta\vec{v} = \vec{v} - \vec{u} = {}^-60 - {}^-75 = {}^+15$, i.e. 15 km h^{-1} east.

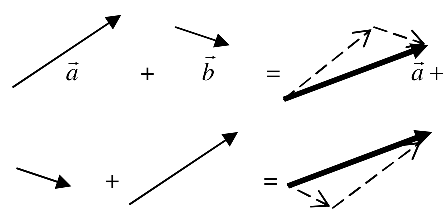
Graphically:

$\Delta\vec{v} = \vec{v} - \vec{u} =$

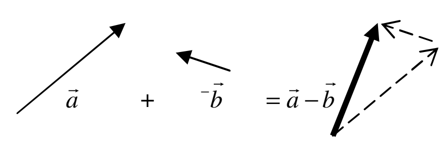


Addition and subtraction of vectors in two and three dimensions

In two and three dimensions, addition is done by placing the head of one arrow to the tail of the other. The order that this is carried out does not affect the resultant.



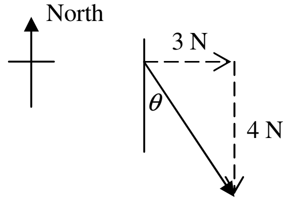
Subtraction is done by changing it to addition first.



Example 4 Two forces, 3 N east and 4 N south act on an object. Find the net force (resultant force) on the object.

The net force is given by the vector addition of the two forces.

Method 1 Draw an accurate scaled diagram, e.g. 1cm : 1 N and measure the length of the resultant vector and its direction.



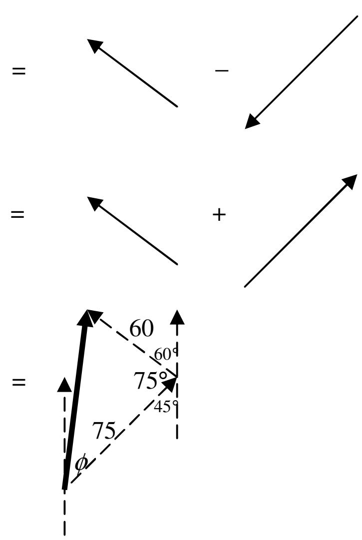
Method 2 Draw a rough sketch and calculate using the trigonometric ratios, the Pythagoras Theorem, the sine or cosine rule.

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 37°$$

$$\therefore \vec{F}_{net} = \sqrt{3^2 + 4^2} = 5 \text{ N S37°E}$$

Example 5 The velocity of a car changes from 75 km h^{-1} SW to 60 km h^{-1} N60°W. What is the change in velocity of the car?

Change in velocity = 60 km h^{-1} N60°W – 75 km h^{-1} SW



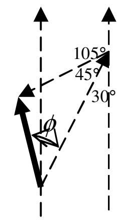
$$|\Delta\vec{v}| = \sqrt{60^2 + 75^2 - 2(60)(75)\cos 75°} \approx 83 \text{ km h}^{-1}.$$

$$\frac{\sin\phi}{60} = \frac{\sin 75°}{83},\ \phi \approx 44°$$

$\therefore \Delta\vec{v} \approx 83$ km h^{-1} N1°E.

Example 6 A car travels 1.2 km N30°E and then 0.7 km S75°W. Find the *displacement* (change in position) of the car from its starting point.

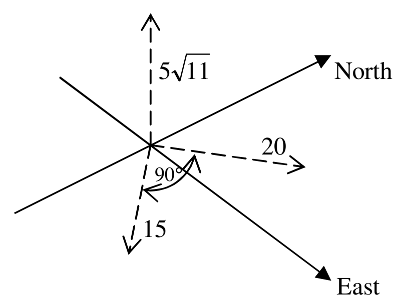
Total displacement $\vec{s}$ from the starting point
= 1.2 km N30°E + 0.7 km S75°W

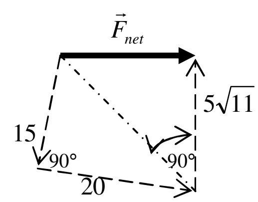


$$|\vec{s}| = \sqrt{1.2^2 + 0.7^2 - 2(1.2)(0.7)\cos 45°} \approx 0.86 \text{ km.}$$

$$\frac{\sin\phi}{0.7} = \frac{\sin 45°}{0.86},\ \phi \approx 35°. \qquad \therefore \vec{s} \approx 0.86 \text{ km N5°W.}$$

Example 7 Three forces, 15 N SE, 20 N NE and $5\sqrt{11}$ N upward act on an object. Find the magnitude of the net force (resultant force) on the object.

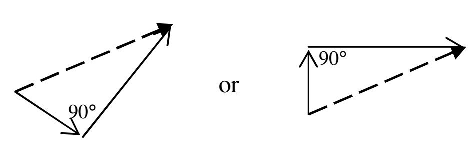




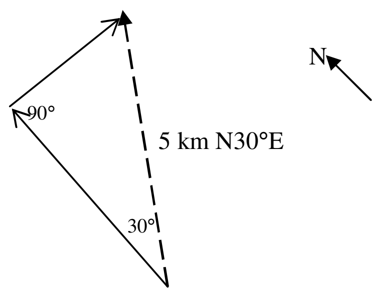
$$F_{net} = \sqrt{15^2 + 20^2 + \left(5\sqrt{11}\right)^2} = 30 \text{ N}$$

Resolving a vector into two perpendicular components

A vector can be decomposed (resolved) into components. In many situations, the most useful way is to resolve a vector into two perpendicular components.



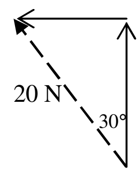
Example 1 A hiker has a displacement of 5 km N30°E. How far to the north and how far to the east is the hiker from her initial position?



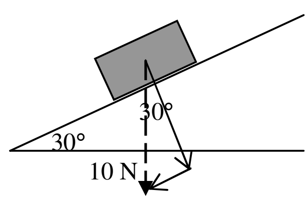
To the north: $5\cos 30° \approx 4.3$ km.
To the east: $5\sin 30° = 2.5$ km.

Example 2 Resolve the 20N force into vertical and horizontal components.

Vertical: $20\cos 30° \approx 17$ N
Horizontal: $20\sin 30° = 10$ N



Example 3 An object slides down a smooth plane inclined at $30°$ to the horizontal. The force of gravity on the object is 10 N. Resolve the force of gravity into two perpendicular components: one parallel to the inclined plane and the other perpendicular to it.



Perpendicular to the plane: $10\cos 30° \approx 8.7$ N.
Parallel to the plane: $10\sin 30° = 5.0$ N.

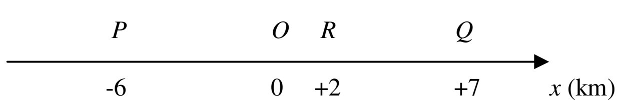
Motion in one dimension

Motion can be described in terms of position, velocity and acceleration. They are vector quantities.

Position

The position of an object is specified in relation to a reference point called the origin. For motion in one dimension, use the number line to indicate positions.

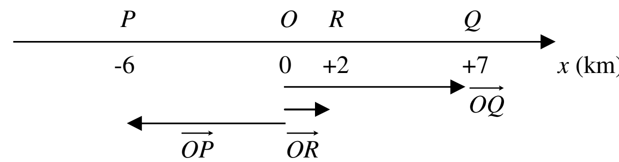
Example 1



Using directed numbers, the position of an object at *P* is -6 km, at *Q* is +7 km, and at *R* is +2 km.

If the number line is running in the east-west direction, the position of an object at *P* is 6 km W, at *Q* is 7 km E, and at *R* is 2 km E.

Using arrows:



Displacement

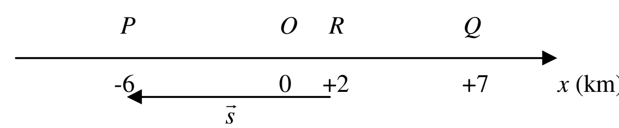
When an object moves its position *x* changes with time *t*. Change in position $\Delta\vec{x}$ is also called displacement $\vec{s}$. It is defined as final position $\vec{x}_f$ – initial position $\vec{x}_i$.

$$\vec{s} = \Delta\vec{x} = \vec{x}_f - \vec{x}_i$$

It is a vector quantity.

Note: Displacement does not depend on the actual path followed by the object. Only the initial and the final positions determine the displacement, e.g. refer to the previous example, if the object moves from *R* through *Q* to *P*, the displacement is $\vec{s} = {}^-6 - {}^+2 = {}^-8$ km or 8 km W.

Graphically, displacement is an arrow of magnitude equal to the straight line distance between initial and final position, and pointing in the direction from initial to final position.



Distance travelled

The distance *d* travelled is the actual length of the path followed by the object. It is a scalar quantity.

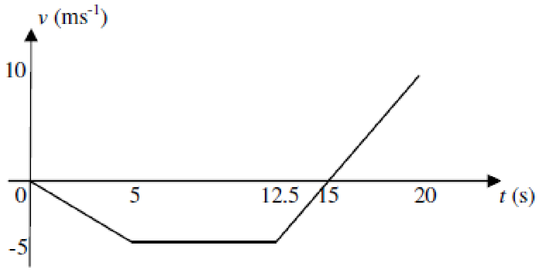
Refer to the previous example, the distance travelled is

$$d = 5 + 13 = 18 \text{ km}.$$

Note: In general, distance travelled does not equal the magnitude of displacement. They are equal only when the object moves in the same direction in the time interval under consideration.

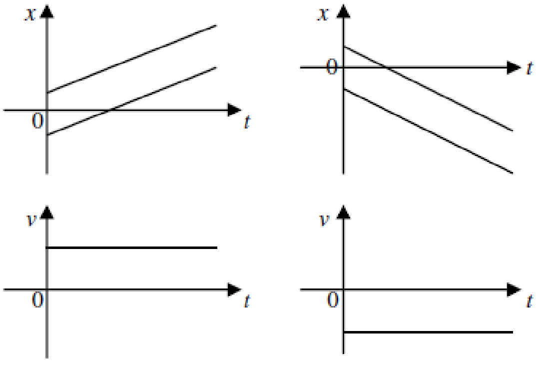
Instantaneous velocity and instantaneous speed.
Instantaneous velocity and instantaneous speed are simply called velocity and speed respectively. The speed of a car is given by the speedometer reading and the velocity is given by the speedometer and compass readings. Therefore, speed is equal to the magnitude of velocity.
The two quantities can be calculated according to the definitions:
Velocity $\vec{v} = \frac{\vec{s}}{\Delta t}$ and speed $v = \frac{d}{\Delta t}$ for $\Delta t \to 0$, i.e. very short time interval.

Velocity-time graph
The motion of an object moving in a straight line can be represented by a velocity-time graph.
Example 1. The following graph shows the velocity of an object at different time. The motion is in the east-west direction. East is chosen to be the positive direction. Describe the motion in terms of velocity, speed, direction.



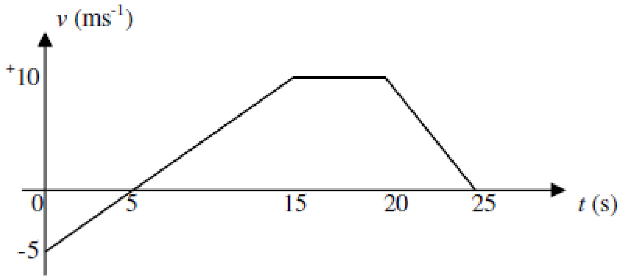
The object starts from rest and travels to the west with its speed increasing uniformly, reaching 5 $m\ s^{-1}$ for the first 5 s. It maintains this velocity (speed and direction) for 7.5 s before slowing down uniformly to a stop momentarily in the next 2.5 s. It then speeds up uniformly to the east in another 5 s, reaching a speed of 10 $m\ s^{-1}$.

Relationship between *v-t* and *x-t* graphs.



Gradient of a position-time graph gives velocity, and area ‘under’ a velocity-time graph gives displacement (not position).

Example 2. From the *v*-*t* graph below, find the average velocity and the average speed. Take north to be the positive direction.



0-5: $s_1 = \frac{1}{2} \times {}^{-}5 \times 5 = {}^{-}12.5\,\text{m}$

5-25: $s_2 = \frac{1}{2} \times (5 + 20) \times {}^{+}10 = {}^{+}125\,\text{m}$

Displacement $= s_1 + s_2 = {}^{+}112.5\,\text{m}$

Average velocity $= \frac{{}^{+}112.5}{25} = {}^{+}4.5$, i.e. 4.5 ms^{-1} N.

Distance travelled $= |s_1| + |s_2| = 12.5 + 125 = 137.5\,\text{m}$

Average speed $= \frac{137.5}{25} = 5.5\,\text{ms}^{-1}$.

Average acceleration and instantaneous acceleration.

Acceleration is the rate of change of velocity. A change in velocity can be the result of a change in speed, direction or both. Acceleration is a vector quantity.

Definitions:

Average acceleration $= \frac{\textit{changeinvelocity}}{\textit{timetaken}}$, i.e. $a_{av} = \frac{\Delta v}{\Delta t}$.

Instantaneous acceleration $a \approx \frac{\Delta v}{\Delta t}$ for $\Delta t \to 0$.

The direction of motion is given by the direction of the *velocity vector*. The direction of the *acceleration vector* indicates the direction of the net force acting on the object.

Speeding up or slowing down

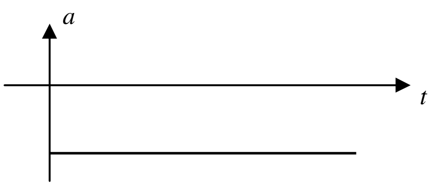
When the velocity and acceleration vectors point in the same direction the object speeds up. When they are opposite in direction, the object slows down.

Speeding up:

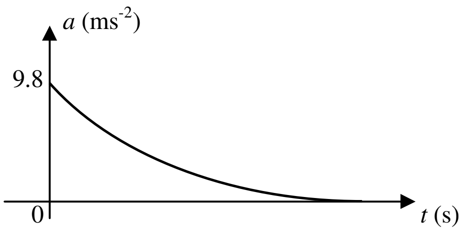
Slowing down:

Acceleration-time graph

Example 1 The following *a-t* graph shows the motion of a ball-bearing projected vertically upwards (taken as the positive direction) under the influence of gravity (assume constant) with negligible air resistance.

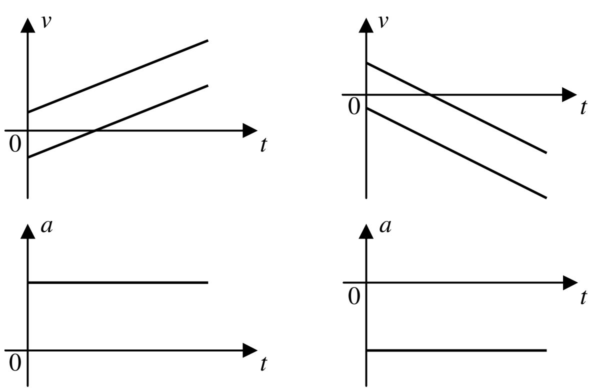


Example 2 The following *a-t* graph shows the motion of a tennis ball dropped from a great height with air resistance. Downward is taken as the positive direction.



When the acceleration reaches zero, the tennis ball falls at constant velocity called its **terminal velocity**.

Relationship between *v-t* and *a-t* graphs



The gradient of a velocity-time graph gives the acceleration, and the area 'under' an acceleration-time graph gives the change in velocity (not velocity).

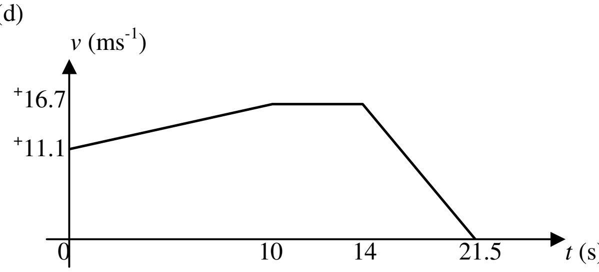
Example 3 Jane travelling east at 40 km h^{-1} increases her speed to 60 km h^{-1} in 10 s.
(a) Calculate her change in velocity in ms^{-1}.
(b) Calculate her average acceleration in ms^{-2}.
Jane travels a further 4.0 s at a constant velocity of 60.0 km h^{-1} east, then slows down to a stop in 7.5 s.
(c) Calculate her average acceleration during the slow down to a stop.
(d) Draw a velocity-time graph for the whole trip, assuming the accelerations are uniform.

(a) Change in velocity= 60 km h^{-1} E – 40 km h^{-1} E = 20 km h^{-1} E ≈ 5.6 ms^{-1} E.

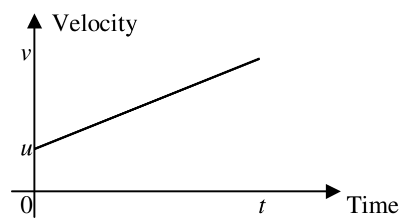
(b) Average acceleration $= \frac{5.6}{10} = 0.56$ ms^{-2} E.

(c) Change in velocity= 0 km h^{-1} E – 60 km h^{-1} E = 60 km h^{-1} W ≈ 16.7 ms^{-1} W.
Average acceleration $= \frac{16.7}{7.5} = 2.2$ ms^{-2} W.

(d)



Motion in a straight line under constant acceleration



u: Initial velocity, i.e. velocity at $t = 0$.
v: Final velocity, i.e. velocity at time *t*.
a: Acceleration (constant).
s: Displacement from the initial position which is the position at $t = 0$.

The gradient of the *v-t* graph gives the acceleration,
i.e. $a = \frac{v-u}{t-0}$, $\therefore v = u + at$ ………(1)
The area under the *v-t* graph gives the displacement,
i.e. $s = \frac{1}{2}(u+v)t$ ………(2)

Eliminating *v* from (1) and (2), $s = ut + \frac{1}{2}at^2$ ………(3)

Eliminating *u* from (1) and (2), $s = vt - \frac{1}{2}at^2$ ………(4)

Eliminating *t* from (1) and (2), $v^2 = u^2 + 2as$ ………(5)

Note: Each equation involves four of the five quantities, *u*, *v*, *a*, *s* and *t*. In these equations *u*, *v*, *a* and *s* are vector quantities, a direction needs to be chosen as positive. These equations are used for motions with constant acceleration. If the acceleration is not constant, they can be used as an approximation by taking the average acceleration as constant acceleration *a*.

Example 1 On a dry road a car with good tyres may be able to slow down at a rate of 4.92 ms^{-1} per second.
(a) How long does it take to come to rest from an initial speed of 24.6 ms^{-1}?
(b) How far does it travel in this time?

Take forward as the positive direction. Since the car slows down, the acceleration vector is opposite to the velocity vector.

(a) $u={}^{+}24.6$ ms^{-1}, $a={}^{-}4.92$ ms^{-2}, $v=0$, find t.

Use $v=u+at$, $0={}^{+}24.6+{}^{-}4.92t$, $t=5.00$ s.

(b) $u={}^{+}24.6$ ms^{-1}, $a={}^{-}4.92$ ms^{-2}, $v=0$, find s.

Use $v^2=u^2+2as$, $0=24.6^2+2({}^{-}4.92)s$, $s={}^{+}61.5$ m.

Distance travelled = 61.5 m.

Example 2 At the instant the traffic light turns green, a car starts with constant acceleration of 2.2 ms^{-1}. At the same instant a truck, travelling at a constant speed of 9.5 ms^{-1}, overtakes and passes the car.
(a) How far beyond the traffic light will the car overtake the truck?
(b) How fast will the car be travelling at that instant?

Take forward as the positive direction. Let T and ^{+}D be the time and displacement when the car overtake the truck

(a) Car: $u=0$, $a={}^{+}2.2$, $t=T$, $s={}^{+}D$.

Use $s=ut+\dfrac{1}{2}at^2$, ${}^{+}D=\dfrac{1}{2}({}^{+}2.2)T^2$ ......(1)

Truck: $u={}^{+}9.5$, $a=0$, $t=T$, $s={}^{+}D$.

Use $s=ut+\dfrac{1}{2}at^2$, ${}^{+}D={}^{+}9.5T$, $\therefore T=\dfrac{D}{9.5}$ ......(2)

Substitute (2) in (1), $D=1.1\left(\dfrac{D}{9.5}\right)^2$, $\therefore D=0$ or 82.

Distance travelled = 82 m.

(b) Car: $u=0$, $a={}^{+}2.2$, $s={}^{+}82$, find $|v|$.

Use $v^2=u^2+2as$, $v^2=2({}^{+}2.2)({}^{+}82)$, $|v|=19$ ms^{-1}.

Free fall vertical motion

When an object moves under the influence of gravity only, it is in **free fall**. Close to the surface of the earth, acceleration due to gravity can be considered as constant and has an approximate value of 9.8 ms^{-2}. The five equations for constant acceleration can be used in free fall vertical motion. The upward direction is usually taken as the positive direction.

Example 1 Raindrops fall to the ground from a cloud 1700m above. If they were not slowed by air resistance, how fast would the drops be moving just before they hit the ground?

Take downward as the positive direction.

$u=0$, $s={}^{+}1700$, $a={}^{+}9.8$, find v.

Use $v^2=u^2+2as$, $v^2=2({}^{+}9.8)({}^{+}1700)$, $|v|\approx 183$ ms^{-1}.

Example 2 A rock is projected vertically upwards from the edge of the top of a tall building. The rock reaches its maximum height 1.60 s after it was launched. Then, after barely missing the edge of the building as it falls downwards, the rock hits the ground 6.00 s after it was launched.
(a) With what upward velocity was the rock projected?
(b) How tall is the building?
(c) What maximum height above the ground was reached?

Take upward as the positive direction.
(a) Consider the upward motion:

$t=1.60$, $v=0$, $a={}^{-}9.8$, find u.

Use $v=u+at$, $0=u+{}^{-}9.8\times 1.60$, $u\approx{}^{+}15.7$ ms^{-1}.

(b) Consider the whole trip:

$t=6.00$, $u={}^{+}15.7$, $a={}^{-}9.8$, find s.

Use $s=ut+\dfrac{1}{2}at^2$, $s=({}^{+}15.7)(6.00)+\dfrac{1}{2}({}^{-}9.8)(6.00)^2$,

$s\approx{}^{-}82.3$. The building is about 82 m tall.

(c) Consider the upward motion:

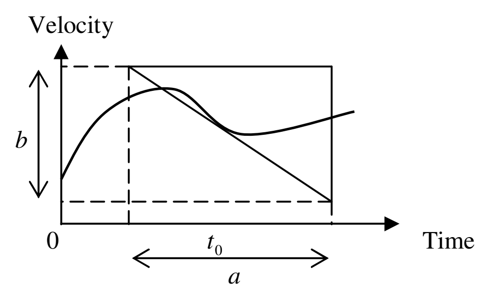
$t=1.60$, $v=0$, $a={}^{-}9.8$, find s.

Use $s=vt-\dfrac{1}{2}at^2$, $s=-\dfrac{1}{2}({}^{-}9.8)(1.60)^2\approx{}^{+}12.5$ m

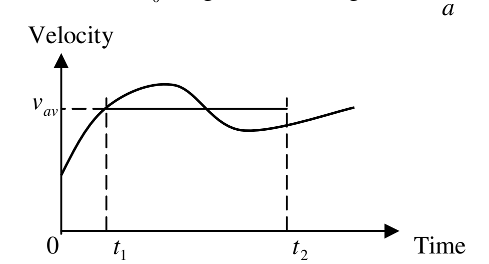
Maximum height reached $=12.5+82.3\approx 95$ m.

Non-uniform motion in a straight line

For motion with changing acceleration, graphical analysis is a handy way to study the motion. If v-t graph of motion is known, gradient of tangent to the graph at a particular time gives the acceleration at that time. Area under the graph between t_1 and t_2 gives the displacement in that interval.



Acceleration (at $t=t_0$) = gradient of tangent $=-\dfrac{b}{a}$

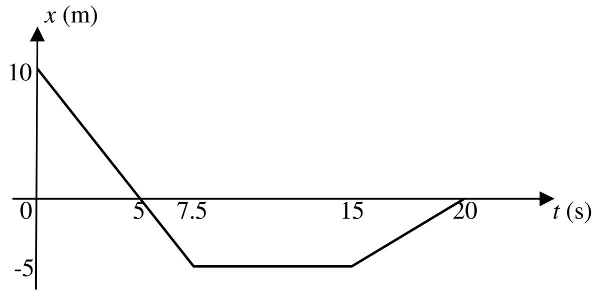


Displacement (between t_1 and t_2) = area under graph $\approx v_{av}(t_2-t_1)$.

Note: v_{av} is the average velocity (by estimation) in the interval between t_1 and t_2.

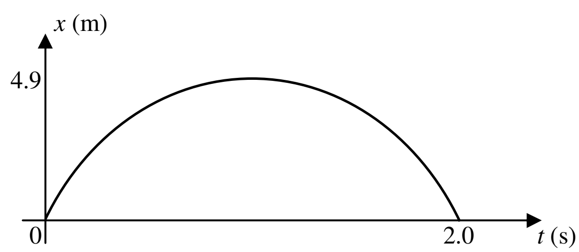
Position-time graph

Example 1 The following graph shows the position of an object at different time. The motion is in the east-west direction. East is chosen to be the positive direction. Describe the motion in terms of positions, directions, displacement and distance travelled.



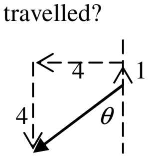
Initially, the object is 10 m east of the origin. It travels 15 m to the west in 7.5 s. Now it is 5 m west of the origin and remains there for another 7.5 s before it turns around and travels 5 m to the east in 5 s, finishing at the origin. Its displacement in the 20 s is -10 m, and the total distance travelled is 20 m.

Example 2 The following graph shows an object in vertical motion. The ground level is chosen as the origin and positions above the ground are considered as positive. Describe the motion in terms of positions, directions, displacement and distance travelled.



The object starts from the ground and moves vertically upwards to a maximum height of 4.9 m in 1.0 s and falls back to the ground in another 1.0 s. The displacement in the 2.0 s is 0 and the total distance travelled is 9.8 m.

Example 3 A person walks 1 km north, 4 km west and then 4 km south.
(a) What is the displacement of the person?
(b) What is the total distance travelled?

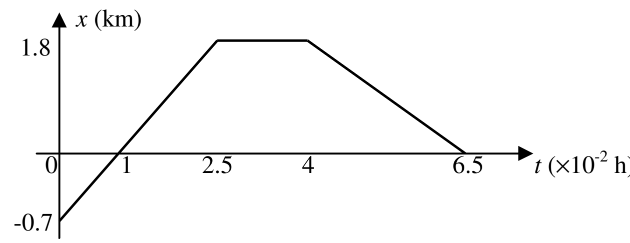


(a) $|\vec{s}| = \sqrt{4^2 + 3^2} = 5$, $\theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 53°$.

$\therefore \vec{s} = 5$ km S53°W.

(b) $d = 1 + 4 + 4 = 9$ km.

Example 4 A motor-cyclist travelled along a straight road running in the NE direction. The following position-time graph shows the motion of the motor-cyclist. NE is taken as the positive direction. A petrol station is the origin *O*.



(a) What was the initial position of the cyclist?
(b) Where was the cyclist at t = 0.065 h?
(c) For how long was the cyclist at rest?
(d) What was the total distance travelled in the first 0.065 h?
(e) What was the displacement in the first 0.065 h?
(f) In which direction did the cyclist travel at t = 0.050 h?

(a) $\vec{x} = {}^{-}0.7$, i.e. 0.7 km SW of the petrol station.

(b) At the petrol station.

(c) 0.015 h.

(d) $d = 0.7 + 1.8 + 1.8 = 4.3$ km.

(e) $\vec{s} = {}^{+}0.7$ km, i.e. 0.7 km NE.

(f) SW.

Average velocity and average speed

Average velocity and average speed are different quantities and defined differently. Average velocity is a vector quantity while average speed is a scalar.

Definitions:

Average velocity = $\dfrac{displacement}{timetaken}$, $\vec{v}_{av} = \dfrac{\vec{s}}{\Delta t}$.

Average speed = $\dfrac{dis\tan cetravelled}{timetaken}$, $v_{av} = \dfrac{d}{\Delta t}$.

Example 1 A car travels on a straight road for 30 km at 60 km h^{-1} and then 80 km h^{-1} in the opposite direction for half of an hour. Find its average velocity and average speed.

Forward: 30 km for 0.50 h.
Opposite direction: 40 km for 0.50 h.
Displacement $\vec{s} = {}^{+}30 + {}^{-}40 = {}^{-}10$ km.
Distance travelled $d = 30 + 40 = 70$ km.
Time taken $\Delta t = 0.50 + 0.50 = 1.0$ h.

$\vec{v}_{av} = \dfrac{{}^{-}10}{1.0} = {}^{-}10$ km h^{-1}. $v_{av} = \dfrac{70}{1.0} = 70$ km h^{-1}.

Note: Average speed is NOT equal to the magnitude of average velocity in general.

Constant Acceleration Formulae

The constant acceleration formulae only apply when the acceleration of the object does not change during its entire motion. The most common example is motion under gravity. The constant acceleration formulae are in the box.

$$v = u + at$$
$$v^2 = u^2 + 2ax$$
$$x = ut + \tfrac{1}{2}at^2$$
$$x = vt - \tfrac{1}{2}at^2$$
$$x = \frac{(u+v)t}{2}$$

x is the displacement
u is the initial velocity
v is the final velocity
a is the acceleration
t is the time period in question
Note that *t* is a time interval, not a specific time.

When using these formulae to solve problems it is best to write down everything that you know from the question, and then write down the thing that you wish to find and then find a formula that relates what you have to what you need. If you cannot find such a formula directly, determine anything you can, and re-read the question to ensure that you have not missed any vital information. Some other facts to consider are:

- t = 0 is the beginning of the time interval being considered, i.e. the instant at which 'u' occurs.
- a negative answer for 't' indicates a time previous to 't' = 0.
- x is not necessarily the same measure as the total distance travelled
- a body that is travelling in one direction and accelerating in the opposite direction is slowing down.
- when given the distance travelled in a certain time interval, this distance is the instantaneous velocity halfway through the time interval. E.g. If a body travels 14 m in the seventh second ('t' = 6 to 't' = 7 sec) then the actual velocity at 6.5 seconds is 14 m/s.
- for motion along the horizontal it is usual to take 'to the right positive' for vector sense
- for vertical motion (bodies projected vertically or dropped from rest) the direction of the initial displacement is usually taken as positive
- for vertical motion, the acceleration (symbolised by 'g') is 10 m/s^2 vertically downwards at all times, even if the body is momentarily at the top of its vertical flight.