Interference diffraction and standing waves

- **investigate and explain theoretically and practically diffraction as the directional spread of various frequencies with reference to different gap width or obstacle size, including the qualitative effect of changing the $\frac{\lambda}{w}$ ratio.**
- **investigate and analyse theoretically and practically constructive and destructive interference from two sources with reference to coherent waves and path difference: nλ and $(n - \frac{1}{2})\lambda$ respectively.**
- **explain the results of Young's double slit experiment with reference to:**
  - **evidence for the wave-like nature of light**
  - **constructive and destructive interference of coherent waves in terms of path differences: nλ and $(n - \frac{1}{2})\lambda$ respectively**
  - **effect of wavelength, distance of screen and slit separation on interference patterns: $\Delta x = \frac{\lambda L}{d}$**
- **analyse the formation of standing waves in strings fixed at one or both ends**
- **explain resonance as the superposition of a travelling wave and its reflection, and with reference to a forced oscillation matching the natural frequency of vibration**

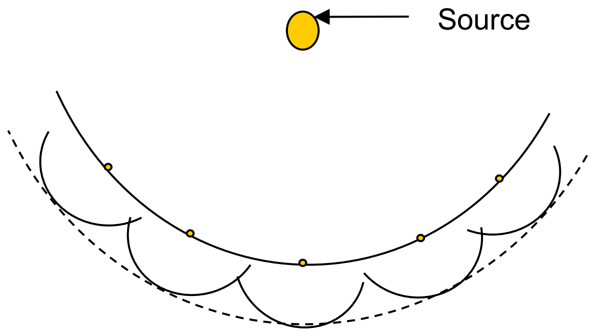
Georges Seurat painted *Sunday afternoon on the island of La Grande Jatte* (in 1884) using a myriad of small coloured dots, in a style now known as pointillism. If you stand close enough to the actual painting (2 m × 3 m) you can see the miniature dots, but as you move away from it, they eventually blend and cannot be distinguished. Moreover, the colour that you see at any given place on the painting changes as you move away.

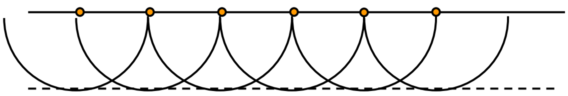
Wave properties

We are going to look at two specific properties of waves, diffraction and interference, and some applications of both. We can observe these wave behaviours, in water waves, with sound and with light.

Huygens Principal of wave propagation

Huygens Principal states that every point on a wavefront acts as a source of a spherical wave. One wavefront is draw. Several points on the wavefront have been identified by orange dots. Each of these points acts as a source of a spherical wave, as shown on the diagram. The next wavefront is the envelope (dotted line) of all of these individual spherical waves. If more points on the original wavefront were included, then the envelope would be even more obvious (and wouldn't have the large gaps between the envelope and the little circles.

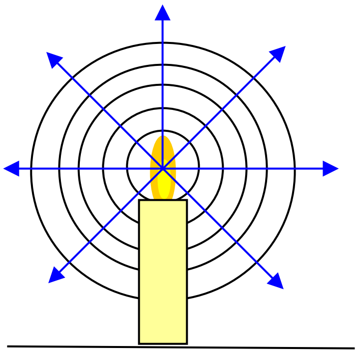


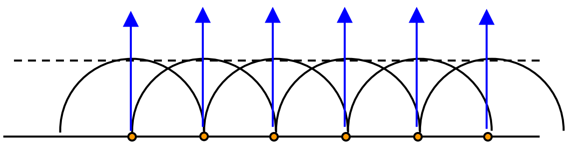


Light that is travelling in a straight line, such as in a torch beam, has straight wavefronts, as shown to the left.

Light Rays

Sometimes we wish to indicate the direction in which light is travelling. It is often cumbersome to draw wavefronts to represent this. A simpler representation is to draw *light rays*. A light ray is drawn perpendicular to the wavefronts. Some *light rays* are drawn on the diagrams below.

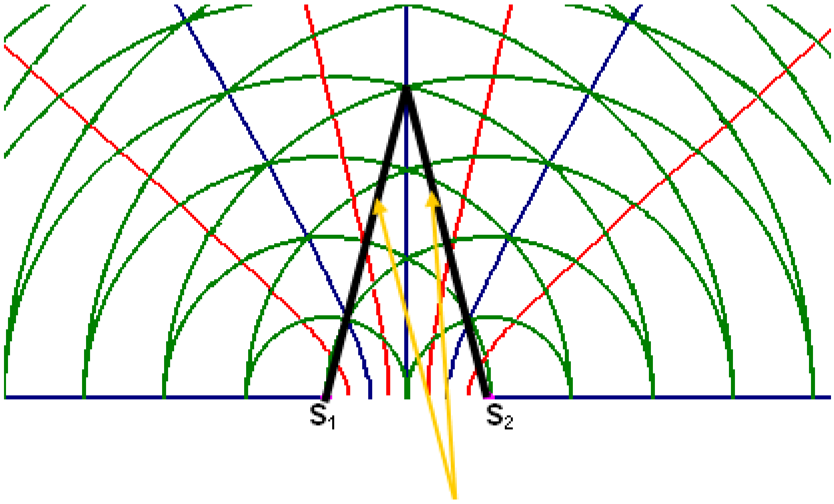




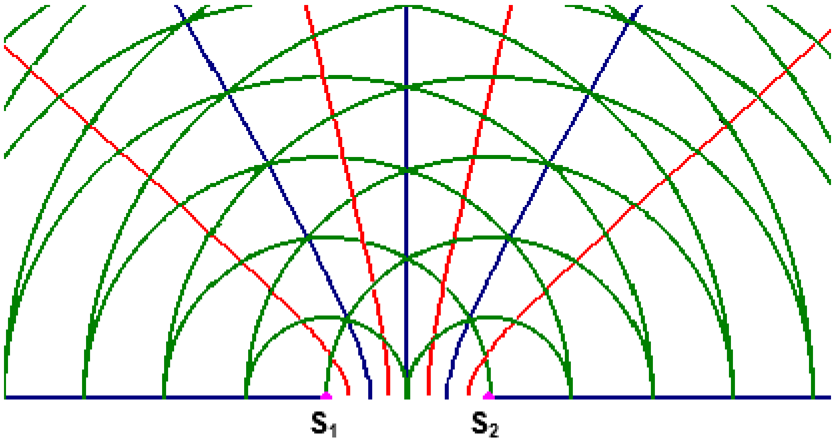
Interference

When waves pass through each other, they can add together so that the reinforce each other; or they can cancel each other out. When the waves add together this is called **constructive interference**, and will lead to a bright point. When the cancel each, this is called **destructive interference** and this leads to a dark point. On the diagram below the blue lines represent the bright spots and the red lines represent the dark areas.

Two dimensional representations



The path difference between these two lines is zero. i.e. they are the same length, both are 4 λ.



The distance from S_1 to P is 3.5 λ while the distance from S_2 to P is 4 λ.
So the difference between these two lengths, called the path difference is 0.5 λ.
Everywhere along the first nodal line (on either side of the central maximum) the path difference will always be 0.5 λ.
If we consider the point **Q,** the path difference is

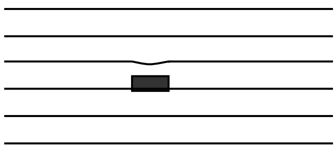
$S_2Q - S_1Q$
$= 5\ \lambda - 3.5\ \lambda$
$= 1.5\ \lambda$.

For any **nodal line** the path difference is summarise by P.D. = (n - ½)λ where n = 1, 2, 3….

For the **anti-nodal lines** the path difference P.D. = nλ where n= 0, 1, 2…..

Diffraction

Diffraction is the bending (flaring) of waves around obstacles in the path of the waves, or as waves pass through narrow openings. In diffraction, the wave remains in the same medium and so its speed, frequency and wavelength are unchanged. The only thing that changes is the direction of the wave as it passes through gaps or around obstacles.

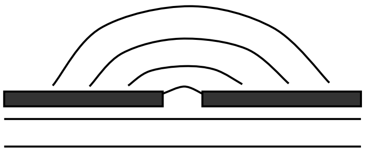


long wavelengths, low frequencies

short wavelengths, high frequencies

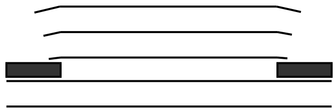
When the obstacle is small compared with the wavelength of the light, there is very little disturbance. Larger 'shadows' occur when the obstacle is much larger than the wavelength of the incident wave. When light travels through a narrow opening the waves bend around both sides of the opening and are diffracted into the region beyond the barriers on both sides.

Circular wavefronts

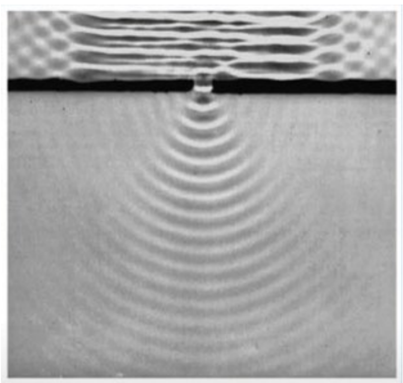


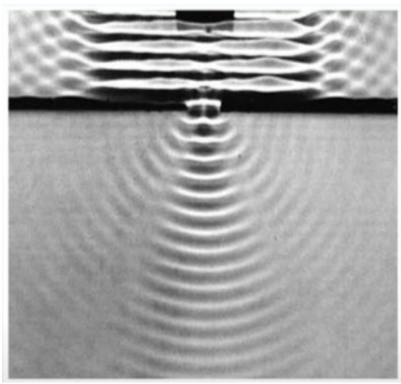
Narrow gap $\lambda > w$

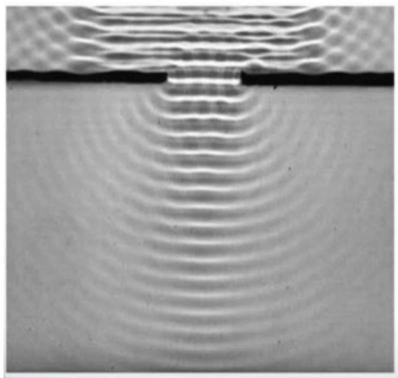
Wavefronts straight, except for slight bending at ends.



large gap $\lambda < w$







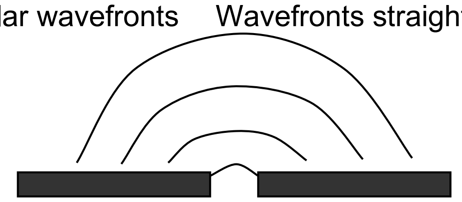
The amount of diffraction (bending) is given by the value of the ratio $\frac{\lambda}{w}$ where w is either the width of the object or the width of the opening.

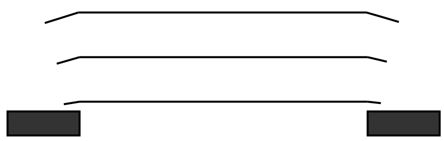
If the ratio $\frac{\lambda}{w} \geq 1$, then it is complete diffraction, i.e. bending through 180^0.

If the ratio $\frac{\lambda}{w} << 1$, there is very little diffraction.

When sound travels through a narrow opening, such as a door, the waves bend around both sides of the opening and are diffracted into the region beyond the barriers on both sides of the doorway. So a narrow gap acts just like obstacles.

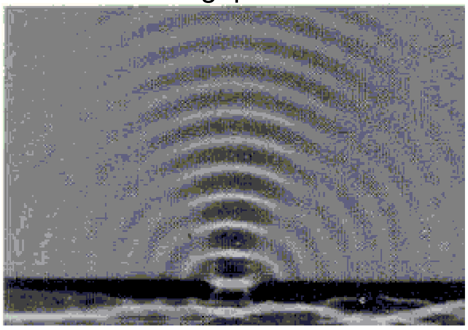
Circular wavefronts Wavefronts straight, except for slight bending at ends.





Narrow gap • $\lambda \geq w$

large gap $\lambda < w$



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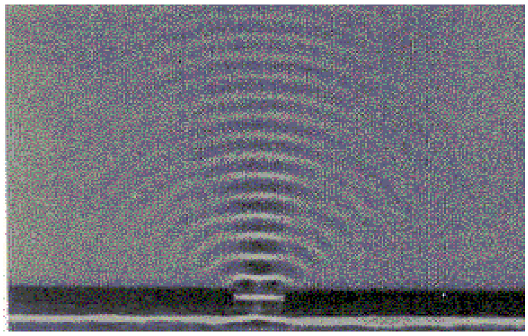
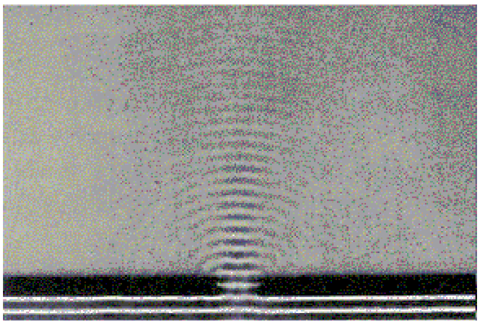


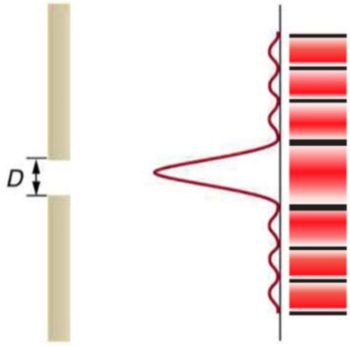
If the ratio $\frac{\lambda}{w} \sim 1$, then it is partial diffraction.



If the ratio $\frac{\lambda}{w} << 1$, then there is very little diffraction, i.e. virtually no bending

Single slit diffraction patterns

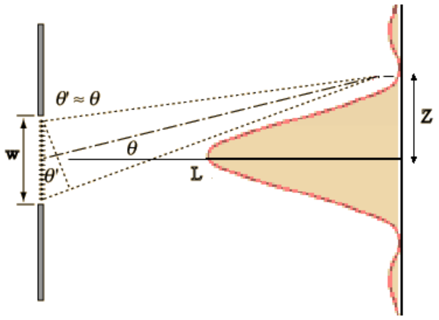
Light can be bent from its ordinary straight-line path by either reflection, refraction or diffraction. When monochromatic light from a distant source (or a laser) passes through a narrow slit the intensity distribution for the diffracted light is shown below.



This pattern is inconsistent with the idea that light is a ray, (if light travelled as a ray, they would form sharp images) and can be explained if light is modelled as a wave. This is evidence of interference of waves.

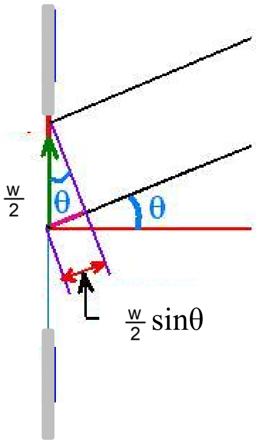
Diffraction is not limited to narrow openings (such as a slit or pinhole). It also occurs when light passes an edge, and in general can be seen for all shadows. The amount of direction depends on wavelength of the wave compared with the size of the obstruction that casts the shadow.

The bright central maximum can be explained by using Huygens wavelets from all points in the slit, that travel about the same distance to reach the centre of the pattern and hence are in phase. It is simplest to consider the other bright fringes to be about midway between adjacent dark fringes.



To determine the position of the dark fringes, we need to make some approximations. If the distance 'L' is much larger than 'w', then the line from the edge of the slit to the screen can be considered to be 'parallel' to the line from the centre of the slit to the screen.

This leads to some more approximations with θ. If the path difference is $\frac{\lambda}{2}$ we will get destructive interference at the screen.



We are also going to approximate that for small values of θ,

$\sin\theta \approx \tan\theta$

$\therefore \frac{\frac{w}{2}\sin\theta}{\frac{w}{2}} = \frac{z}{L}$

$\therefore$ the first node is when $\frac{\lambda}{2} = \frac{w}{2}\sin\theta$

$\therefore w \sin\theta = \lambda$ for the first node.

It can be shown that the m^{th} node is given by:

$w \sin\theta = m\lambda$, for m = 1, 2, 3 …. (minima – dark fringes)

Effect of wavelength

As the wavelength is increased the pattern spreads out.

Small wavelength

Med. wavelength

Large wavelength

Effect of the gap size

As the gap size is increased the number of lines increases.

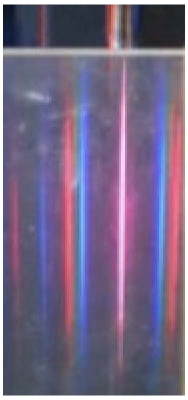
Small Gap

Med. Gap

Large Gap

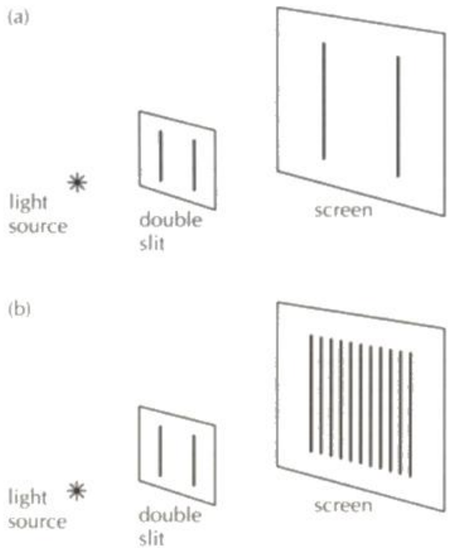
Diffraction pattern for Helium

Diffraction pattern for Mercury



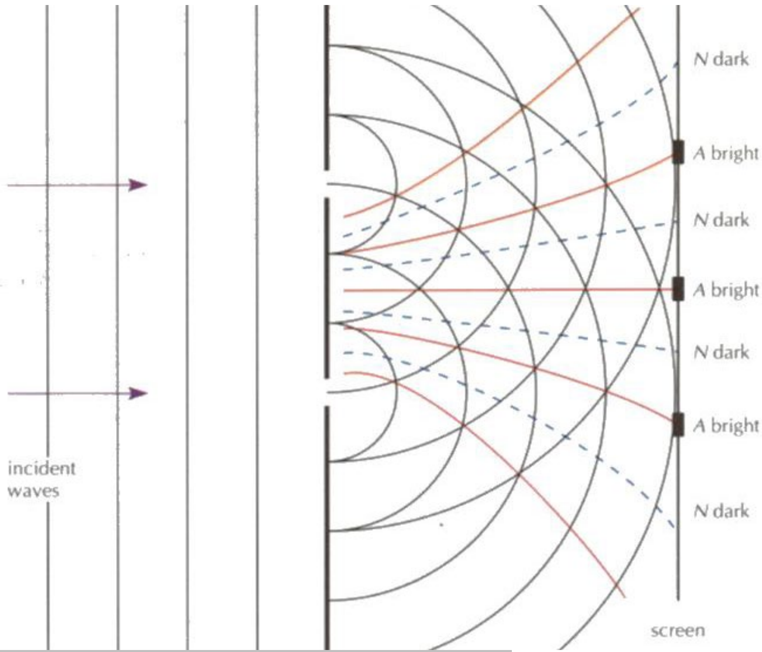
Young's Double slit experiment

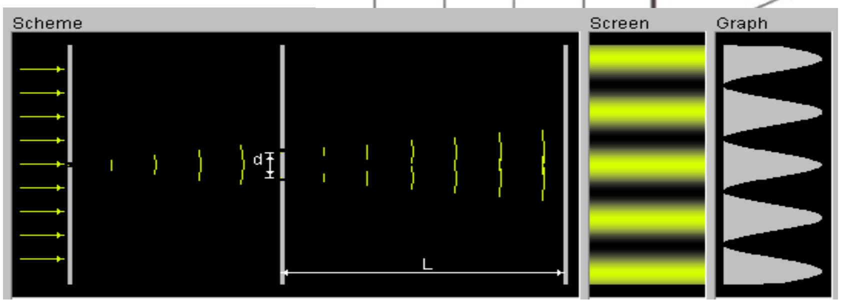
In 1801, Thomas Young, revived the wave model with an experiment that demonstrated the wave nature of light. He allowed the light from a distant source to pass through two narrow parallel slits with a screen placed some distance behind them. The particle model of light predicts that two bright lines will be formed on the screen, one for each slit. In fact, Young actually observed a series of alternating bright and dark lines or fringes.



Young explained this result, using Huygens' principle and assuming that each narrow slit acted as a source of secondary waves which spread out behind the slits and interfered with each other to form the bright and dark bands. Antinodes where crests met crests and troughs met troughs and constructively interfered with each other to form the bright lines. Nodes were formed where crests met troughs and troughs met crests and the displacements cancelled each other out by deconstructive interference, producing lines of minimum intensity.

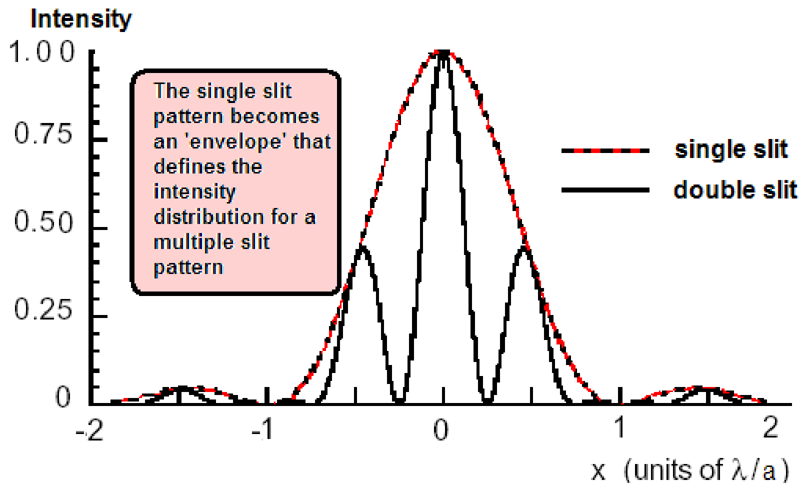
This experiment showed that light would produce an interference pattern, because it diffracted when passing through very small slits if the sources were close enough together. The pattern produced has a pattern of nodes and antinodes just like sound or water. A series of light and dark lines were observed on the screen. Dark lines correspond to cancellation, or nodes, bright lines to antinodes.





The pattern can be described algebraically as $x \approx \frac{n\lambda L}{d}$. Where x is the distance between the central maximum and the local maximum, n is the nodal line, λ is the wavelength, L is the perpendicular distance from the slits to the screen, and 'd' is the distance between the two slits.

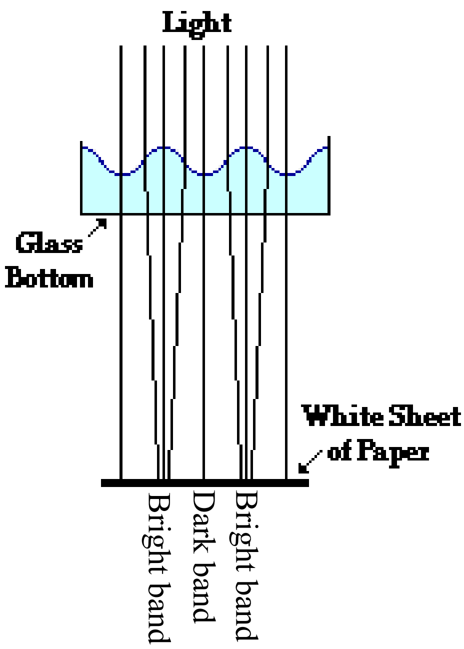
Single slit/Double slit patterns.

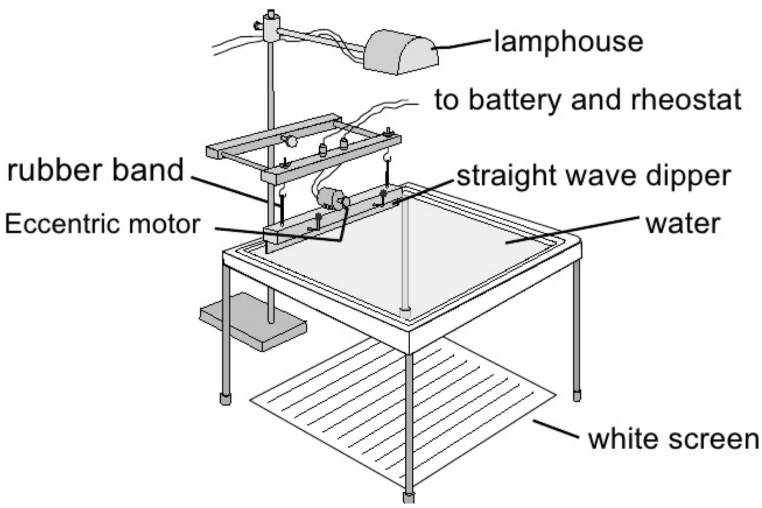


Interference is a wave phenomenon and so can also be observed with sound waves and water waves.

Ripple tanks

Water waves are often investigated using ripple tanks.
They are used to demonstrate wave behaviour. When light is shone from above, the image of the waves is shown on the screen below the tank.

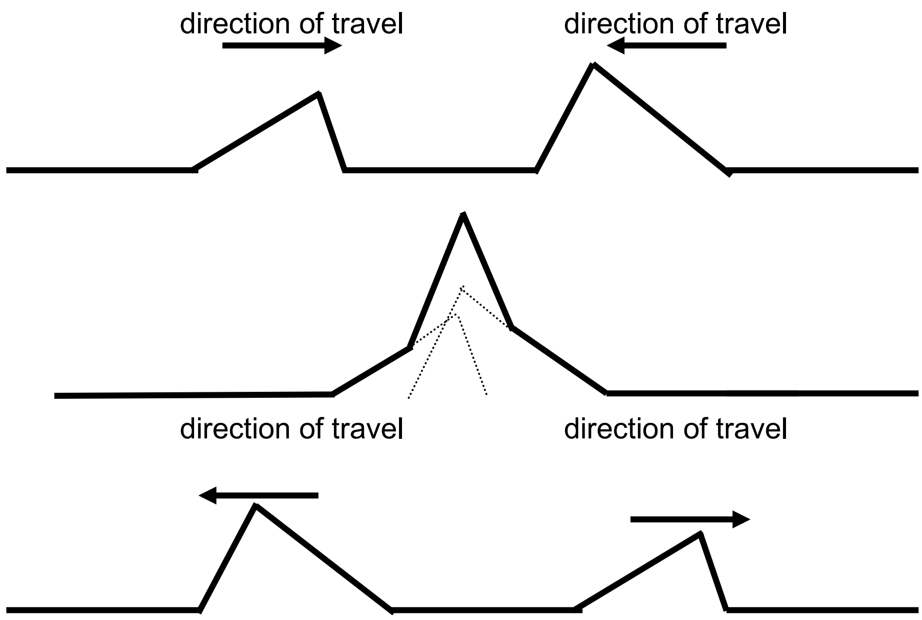




The crests act as a converging lens and create a bright line whilst the troughs act as a diverging lens and create a dark band.

Superposition

When different waves pass through the same region of space, the individual waves add together to produce the resultant sound wave. This is called superposition.
The displacement of two waves combing with each other is calculated by the vector addition of the two components. The displacement of the combined pulse is the sum of the separate displacements. The two pulses pass through each other without being altered.
Constructive interference is when the two pulses pass through each other and superimpose and reinforce each other to give a maximum disturbance of the medium.
Destructive interference is when the two pulses pass through each other and superimpose and cancel each other out to give minimum or zero disturbance of the medium.
When considering sound waves, constructive interference produces louder sounds and destructive interference results in a quieter sound.



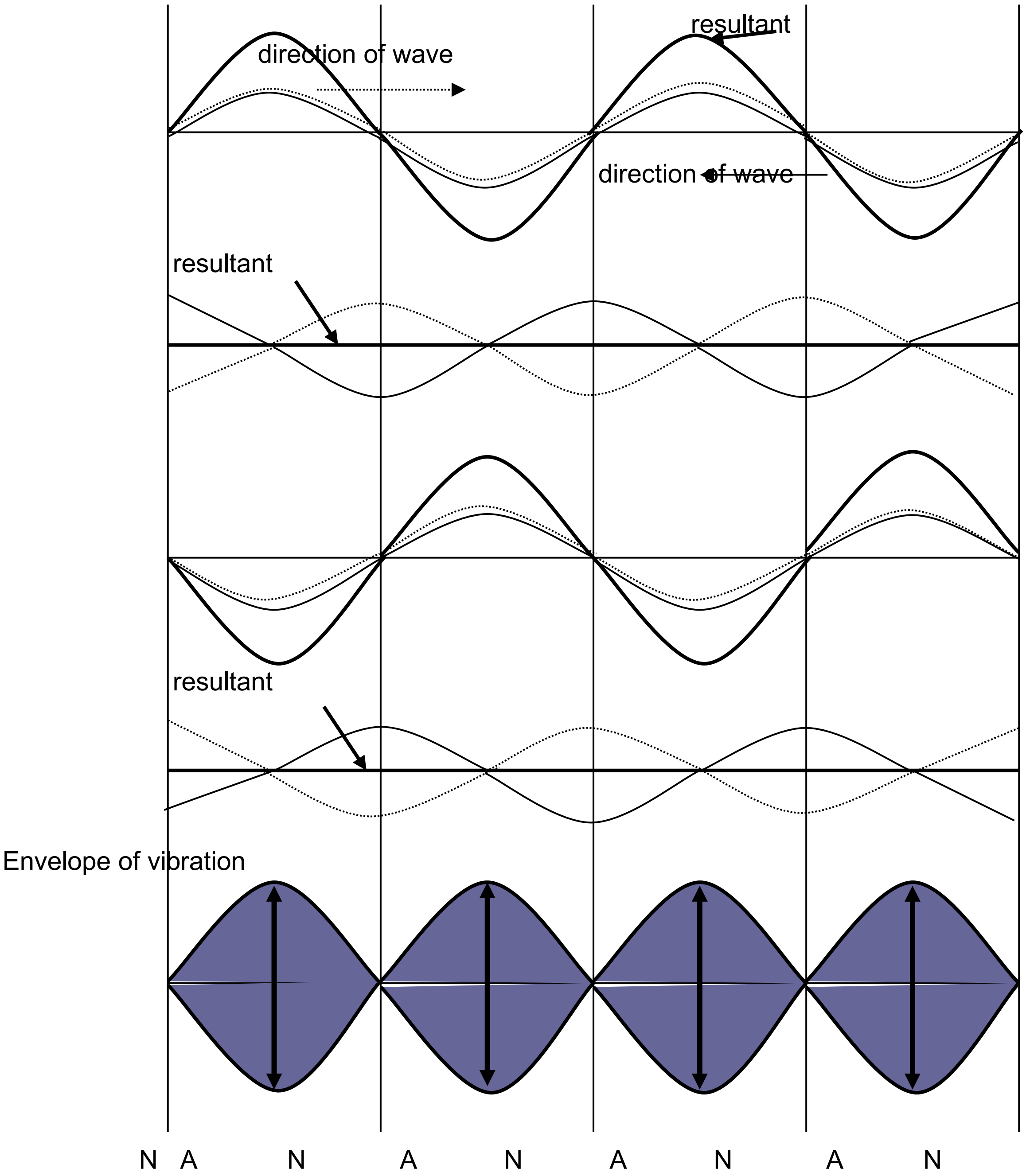
Standing waves

If we have two identical waves travelling in the opposite directions in one medium we get a **standing** or **stationary** wave. The superposition principle is used to obtain the waveform.

- Certain points marked N = node.
- Loops or antinodes, marked A, midway between the nodes.

The wave does not progress through the medium.

- Wavelength is the same as that of the components.
- Maximum amplitude of the resultant wave is twice that of the components.
- The distance between adjacent nodes or antinodes is $\frac{\lambda}{2}$.
- They can only be produced by the superposition of two waves of equal amplitude and frequency travelling in the opposite direction
- They are the result of resonance and occur only at the natural frequencies of the vibration.



Reflection in strings

When a wave reaches a free end, or yielding boundary, it will reflect with crests as crests and troughs as troughs. Strings in musical instruments are always fixed at both ends.

The wavelength of the standing waves corresponding to the natural harmonics is $\lambda_n = \dfrac{2L}{n}$ or $f = \dfrac{nv}{2L}$

All harmonics (n = 1,2,3, …) may be present, the ratio of frequencies $f_1 : f_2 : f_3 = 1 : 2 : 3$.

If the string is only fixed at one end, then only the odd harmonics are available

It can be shown that the fundamental frequency of a stretched wire depends on 3 things: length, tension and mass per unit length. Combining these gives a single formula from which the fundamental frequency of a stretched wire can be found:

$$f = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

where:

f = fundamental frequency in Hertz (Hz)

L = length in metre (m)
T = tension in Newton (N)
m = mass per unit length in kg/m.

Often T and m are constant so $f_2L_2 = f_1L_1$ and when L and m are constant $\frac{f_2}{\sqrt{T_2}} = \frac{f^1}{\sqrt{T_1}}$

If we combine $v = f\lambda$ and $\lambda = \frac{2L}{n}$ we can get $v = \sqrt{\frac{T}{m}}$ where v is the velocity of propagation (m s^{-1}).

Reflections in tubes

Wind instruments rely on reflection of sound waves. When a compression arrives at the end of the tube this high pressure this will reflect back down the tube resulting in a compression being reflected as a rarefaction. A rarefaction arriving at an open end creates an area of low pressure. This will draw free air from beyond the end into the tube creating an area of higher pressure, ie a compression

Wind instruments and air columns (forced vibrations)

Many instruments produce sound from the vibrations of standing waves in a column of air within the tube or pipes of the instrument eg. woodwinds, brasses and the pipe organ. Amplification of the sound comes through resonance within the air column. Sometimes a vibrating reed or the performer’s lips set up the initial vibration in the air column. In the flute and organ, air is directed over an opening, creating turbulence to set up the vibration.

The air in the tube vibrates with a variety of frequencies but only certain frequencies persist. These correspond to the standing waves established in the tube. The lowest frequency standing wave is called the fundamental.

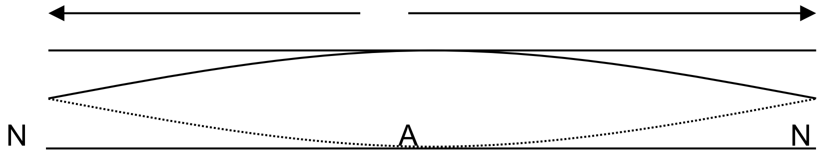
Pressure variation in pipes (Open at both ends)

First harmonic (fundamental frequency)

L

$\lambda_1 = 2L$

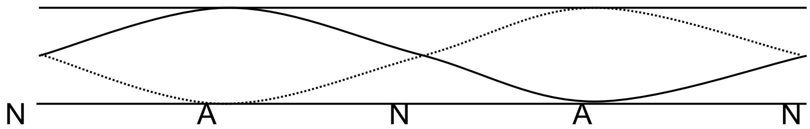
$f_1 = \dfrac{v}{2L}$



Second harmonic (first overtone)

$\lambda_2 = \dfrac{2L}{2} = L$

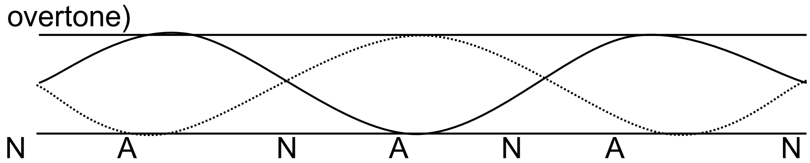
$f_2 = \dfrac{2v}{2L} = 2f_1$



third harmonic (second overtone)

$\lambda_3 = \dfrac{2L}{3}$

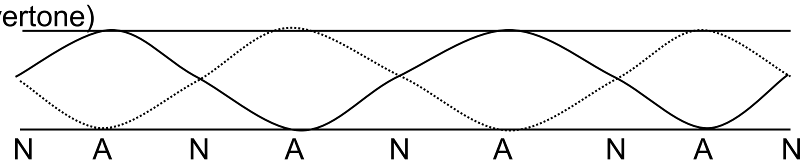
$f_3 = \dfrac{3v}{2L} = 3f_1$



fourth harmonic (third overtone)

$\lambda_4 = \dfrac{2L}{4} = \dfrac{L}{2}$

$f_4 = \dfrac{4v}{2L} = 4f_1$

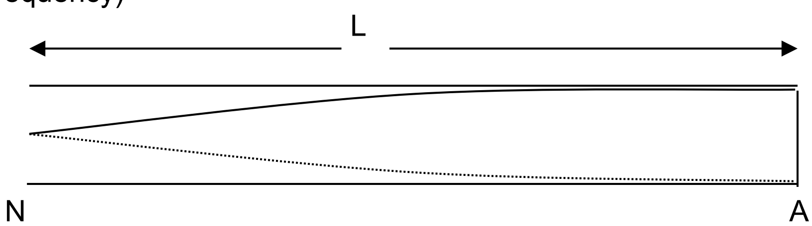


Pressure variation in pipes (Closed at one end)

First harmonic (fundamental frequency)

$\lambda_1 = 4L$

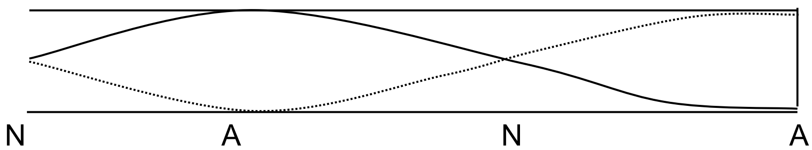
$f_1 = \dfrac{v}{4L}$



third harmonic (first overtone)

$\lambda_2 = \dfrac{4L}{3}$

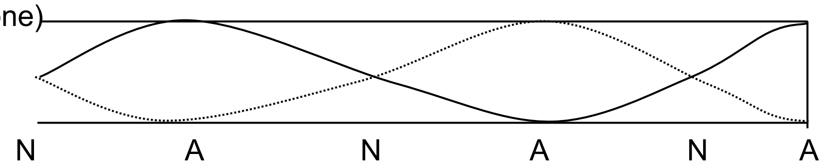
$f_2 = \dfrac{3v}{4L} = 3f_1$



fifth harmonic (second overtone)

$\lambda_3 = \dfrac{4L}{5}$

$f_3 = \dfrac{5v}{4L} = 5f_1$



The term *overtone* is applied to harmonics other than the fundamental frequency.

Summary

The main features of standing waves in air columns can be summarised

- at open ends of the tube there is always a node
- at closed ends there is always an antinode
- the wavelength of the sound must fit the length of the pipe. The length of the air column that vibrates is slightly longer than the length of the pipe, since nodes form just outside the end of the pipe
- the fundamental vibration in a closed pipe has a wavelength twice as long as the fundamental in an open pipe of the same length. This makes the frequency of the sound produced by a closed pipe half that of the same length of an open pipe, so it is an octave lower.

Resonance

Blowing air across the mouth of a bottle causes the air inside to vibrate and a sound is produced.
Resonance occurs when a forcing frequency, the same as the natural frequency, is applied.
Eg. The oscillations of a person on a swing have a certain natural frequency. The amplitude of these oscillations will decrease unless the lost energy is replaced. To keep the swing moving it is best to be pushed at exactly the right times. The frequency of the push must be the same as the natural frequency to get the best response. This is called resonance.