GRAVITATIONAL FIELD NOTES and
HOW TO SOLVE PROBLEMS WITH GRAVITY

Any two objects that have mass will exert a gravitational force of attraction. Gravitational forces are very weak and only become noticeable when at least one of the objects is extremely massive. Gravitational field problems could be divided into 3 groups – questions requiring algebraic manipulations with law of gravity, questions on calculations of work done against/by gravity using graphs and weightlessness explanation questions.

Algebraic manipulations with law of gravity.

To solve those problems students need to have a look on the next set of formulas (**must** have them on the cheat sheet)

$$F = G\frac{mM}{r^2} \quad (1)\text{(Newton's law of gravity)}$$

$$F = mg \quad (2)$$

$$T = \frac{2\pi r}{v} \quad (3)$$

$$F = \frac{mv^2}{r} \quad (4)$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM} \quad (5)(Kepler's\ law)$$

Rearranged (5) $T = \sqrt{\frac{4\pi^2 r^3}{GM}}\ or\ r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$

$$v^2 = \frac{GM}{r} \quad (6)$$

$$g = \frac{GM}{r^2} \quad (7)$$

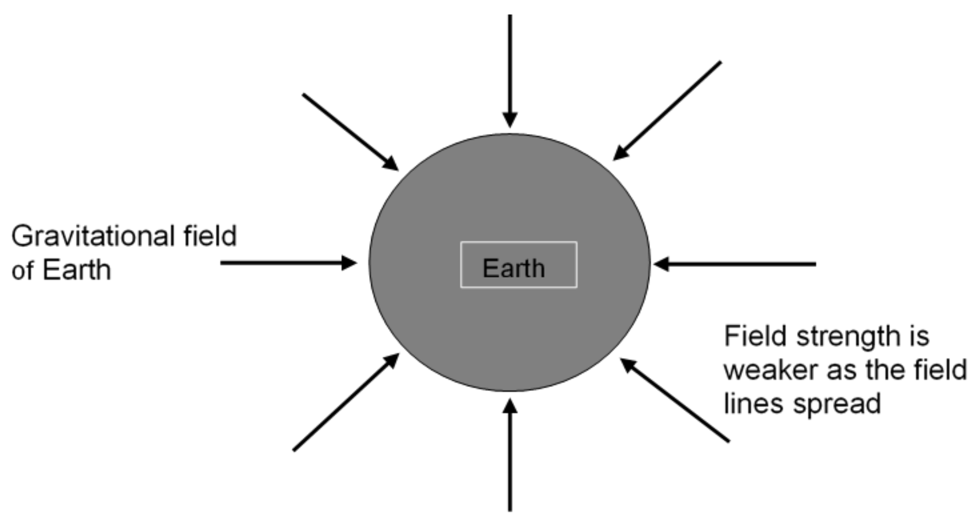
Combining (2) and (6): $T = 2\pi\sqrt{\frac{r}{g}}$ (8),

where G is universal gravitational constant ($G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$, M is the mass of the object in the center, m is the mass of the object which is orbiting around M, T is the period of rotation, v is the speed, g is the strength of gravitational field, r is the radius of the orbit (measured from the centre of the mass in the center!, so if height above the surface is given, don't forget to add radius of the object). Identify which formula is connecting one unknown quantity with known, substitute values, rearrange if necessary and do calculations.

Four important points:

- period, speed and strength of gravitational field are independent from the mass of rotating object;
- student **must** show the substitution of the numbers into the appropriate formula (otherwise minus 1 mark)
- calculations should be performed with cautions as students tend to make a mistake when substituting many numbers with big powers of 10. So it will be wise to calculate in 2 steps, numbers themselves and powers separately. (Example: $g = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(6.4 \times 10^{6})^2}$ becomes $g = \frac{6.67 \times 6.0}{(6.4)^2} \times \frac{10^{-11} \times 10^{24}}{(10^{6})^2}$ resulting in 0.977×10^1, which is 9.8.
- There are several common errors that students make doing these calculations. The most common is to forget to square the bottom line, the next most common is to not use sufficient brackets on your calculator to get the calculation correct.

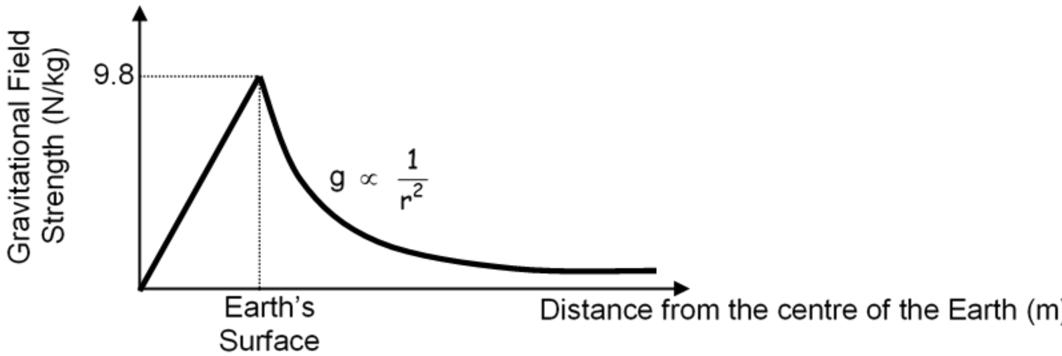
Gravitational fields shape



The force between the two bodies = weight of small body $G\frac{mM}{r^2} = mg$.

Thus the acceleration due to gravity at the surface of the earth, g, is determined by the mass of the earth and the radius of the earth, **<u>not</u>** the size of the mass in the field.

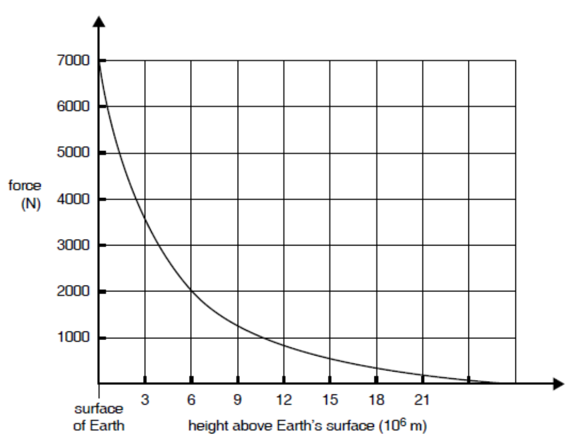
Any object that is falling freely through a gravitational field will fall with an acceleration equal to the gravitational field strength at the point. At the surface of the Earth, the gravitational field strength, g, is 9.8 Nkg^{-1}, it becomes weaker further from the Earth.



If a satellite is in a geo-stationary orbit, the satellite orbits the Earth every 24 hours (period of rotation equals the period of the Earth rotation around its axis). This is a 'synchronous' orbit and keeps the satellite above the same place all the time. Such orbit is possible only above equator and direction of the rotation should be the same as the direction of the Earth rotation.

Questions on calculations of work done against/by gravity.

In these questions students need to calculate area under the graph of force (gravitational field strength) against distance from the center of the object. You should carefully look what is on the vertical axis – force or field strength as if it is field strength you must multiply result obtained from the graph by the mass of space ship.



Area should be calculated by identifying area of one square (don't forget the powers of the units on the axis!), counting the squares between required distances and multiplying those results. Of course calculations will be much more precise by antidifferentiation of the formula (1) or (7), but physics course designed without calculus involved. Obtained result will give work done by gravity (if space ship approaching the object) or against gravity (if it moving away) and will give change of gravitational potential energy.

Sometimes scale on the axis are such that this graph looks very similar to the straight line and thus shape under the graph looks like trapezium so formula for the area of the trapezium could be used. But it is better (more safe) to count the squares as for example in 2018 students who used area of the trapezium lost 1 mark.

If question asking about energy required to put satellite in the orbit of height h above the planet, the area should be calculated from the radius of the planet to r+h. But if you will give satellite just this energy, it will fall back, so kinetic energy required on top of that. Required kinetic energy can be found using formula (6).

If question asking about the speed of the space ship when it approaching the planet you need to write **total** energy of the ship at the starting point of observations.

$$E = E_{k0} + E_{p0} = \frac{mu^2}{2} + E_p$$

and equalize it to the **total** energy at the end point of observations.

$$\frac{mv^2}{2} = \frac{mu^2}{2} + (E_{p0} - E_p)$$

Where $E_{p0} - E_p$ is the area under the graph.

Weightlessness.

True weightlessness is when gravitational field equal to 0. It can be observed in 2 cases – when object is in the deep space far away from any massive bodies or object is between 2 bodies in the point where gravitational fields cancel each other, but it will be only single point.

In all other cases there is an apparent weightlessness. Apparent weightlessness means that normal reaction force is equal to 0, so object is in a free fall and it’s acceleration equal acceleration due to the gravity. Object is not truly weightless in this case as gravity is still acting on it.