Forces and Motion

Study Design

- **model the force due to gravity, *Fg*, as the force of gravity acting at the centre of mass of a body, *Fg* = *mg*, where *g* is the gravitational field strength (9.8 $N\ kg^{-1}$ near the surface of Earth)**
- **model forces as vectors acting at the point of application (with magnitude and direction), labelling these forces using the convention 'force on A by B' or *F*on A by B = –*F*on B by A**
- **apply Newton's three laws of motion to a body on which forces act: $F_{net} = ma$, *F*on A by B = –*F*on B by A**
- **apply the vector model of forces, including vector addition and components of forces, to readily observable forces including the force due to gravity, friction and reaction forces**
- **calculate torque: $\tau = Fr_{\perp}\ or\ \tau = F_{\perp}r$**
- **investigate and analyse theoretically and practically translational forces and torques in simple structures that are in rotational equilibrium**
- **investigate and apply theoretically and practically Newton's three laws of motion in situations where two or more coplanar forces act along a straight line and in two dimensions.**

Types of forces

Forces can be divided into two major categories, field forces and contact forces. Forces that act at a distance are called FIELD FORCES, (gravitational, electrical, magnetic, strong or weak nuclear).

The relationship between a force and the acceleration it causes was first understood by Isaac Newton (1672 – 1727). Newton summarised all motion by three laws:

<u>Newtons 1st law of motion</u>

If an object has zero net force acting on it, it will remain at rest, or continue moving with an unchanged velocity.

A body at rest will remain at rest unless acted on by a net force.
A body in motion will remain in motion with uniform velocity unless acted on by a net force.
This is also known as Inertia. Inertia is a body's tendency to remain in its current state. A body at rest will tend to remain at rest, a body in motion will tend to remain in motion. The inertia of an object is related to the mass of the object, so the more mass the more inertia.

This is the reason that you need to wear a seatbelt in the car. You could be a body in motion that will continue in motion through the windscreen unless a net force acts on you (your seatbelt).
An important consequence of this law was the realisation that an object can be in motion without a force being constantly applied to it. When you throw a ball, you exert a force to accelerate the ball, but once it is moving, no force is necessary to keep it moving. Prior to this realisation it was believed that a constant force was necessary, and that this force was supplied by that the air pinching in behind the ball. This model, first conceived by Aristotle, proved tenacious, and students still fall into the trap of using it.

Newton's first law is commonly tested on the exam. This is achieved by the inclusion of statements such as "*An object is moving with a **constant velocity***" within questions. Whenever you see the key words ***constant velocity*** in a question, you should highlight them. The realisation that the object is travelling at a constant velocity, and hence that the net force on the object is zero, will be essential for solving the problem.

Newtons 2nd law of motion

This law relates to the sum total of the forces on the body ($\mathbf{F}_{net}$) the body's mass (m) and the acceleration produced ($\mathbf{a}$)

$$\mathbf{a} = \frac{\mathbf{F}_{net}}{m}$$

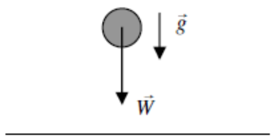
this is commonly written as $\mathbf{F}_{net} = m\mathbf{a}$.

Note $\mathbf{F}_{net}$ must have the same direction as '**a**'.

In words, Newton's Second Law states that a force on an object causes the object to *accelerate* (change its velocity). The amount of acceleration that occurs depends on the size of the force and the mass of the object. Large forces cause large accelerations. Objects with large mass accelerate less when they experience the same force as a small mass. The acceleration of the object is in the same direction as the net force on the object.

Mass and Weight

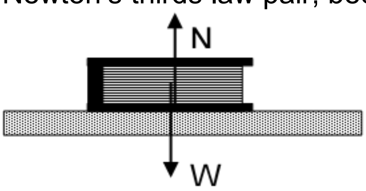
Mass and **weight** are different quantities. Mass is defined as the amount of matter in a body, it is measured in kilograms. It is sometimes referred to as inertia, which is the tendency for an object to resist acceleration. Since $\mathbf{F} = m\mathbf{a}$, a larger force is needed to give a larger mass the same acceleration.
Weight (W) is not the same as mass. Weight is the force of gravity on an object. The acceleration due to gravity is 'g', so the weight of a mass 'm' is given by $\mathbf{F} = m\mathbf{g}$ (Newtons). The amount of matter in an object does not change, hence the mass of an object is always the same. However, 'g' varies from place to place on the earth's surface depending on the distance from the centre of the earth, so weight does change. On the surface of Earth g = 9.8 m s^{-2}.
W and **g** are both vectors pointing towards the centre of the earth, i.e. downwards.



Newton's Third Law

For every action there is an equal but opposite reaction.
When an object applies a force to a second object, the second object applies an equal and opposite force to the first object. By this we mean that forces always exist in pairs, an object cannot push on a second object without the second object pushing back on it. An example of this could be a girl pushing on a wall, and the wall pushing back on the girl.
This can be summarised as:
$\text{Force}_{(\text{on A by B})} = -\text{Force}_{(\text{on B by A})}$

This law is the most commonly misunderstood. You need to appreciate that these action/reaction forces act on **DIFFERENT OBJECTS** and so you do not add them to find a resultant force. For example, consider a book resting on a table top as shown in the diagram below. There are two forces acting on the book: Gravity is pulling the book downward and the tabletop is pushing the book upwards. These forces are the same size, and are in opposite directions but **THEY ARE NOT** a Newton's thirds law pair, because they both act on the same object.



In the example of the book on the table the **force** $_{\text{on table by book}}$ is a Newton third law pair with the **force** $_{\text{on book by table}}$. Notice the first force is on the book and the second force is on the table. They do not act on the same object. Similarly the weight force, which is the gravitational attraction of the earth on the book, is a Newton third law pair with the gravitational force of the book on the earth. The gravitational effect of the book on the earth is not apparent because the earth is so massive that no acceleration is noticeable.

Force as a vector

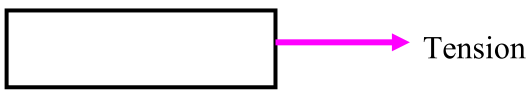
A force exists whenever the shape of an object or the motion of an object changes. The size of the force can be determined by how much the shape or the motion of an object is changed. Changing the motion of an object implies accelerating it. The force is found by multiplying the mass of the object by the acceleration that the force causes. Force is a vector. $\mathbf{F} = m\mathbf{a}$

This is much better thought of as $\mathbf{a} = \frac{\mathbf{F}}{m}$

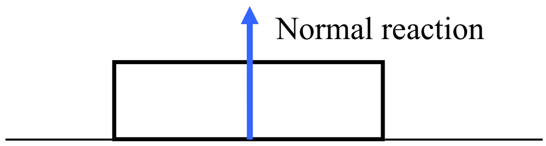
Where the acceleration is in m/s^2, the mass is in kilograms and the unit of force is Newton (N).
A force of 1 Newton will accelerate a 1 kg mass at $1 m/s^2$.

Contact forces

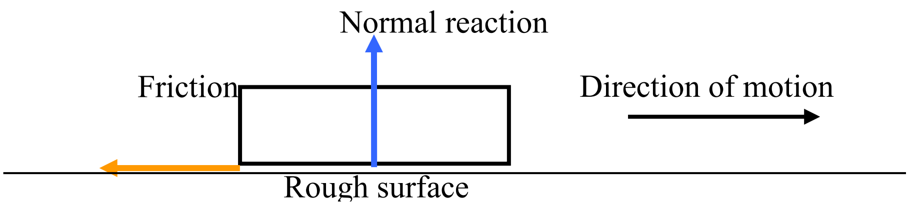
(i) **Tension** (in a rope, chain etc.) **always** act away from the body. (Can you push something with a string??)



(ii) **Normal reaction** (of a surface). Whenever a body is in contact with any surface, that surface exerts a force on the body and this force acts perpendicularly to and away from the surface. If **N** is the only force the surface exerts on a body moving across it, then, the surface is said to be smooth.



(iii) **Friction.** Some surfaces (called rough surfaces) exert two reactions on a body moving over it, - the normal reaction **N**, and a frictional force **F**, which always opposes motion.



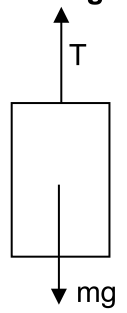
Drawing Force Diagrams

You will often be asked to draw diagrams illustrating forces. There are several considerations when drawing force diagrams:

- The arrows that represent the forces should point in the direction of applied force. The length of the arrow represents the strength of the force, so some effort should be made to draw the arrows to scale.
- An arrow representing a *field force* should begin at the centre of the object.
- An arrow representing a *contact force* should begin at the point on contact where the force is applied.
- All forces should be labelled.

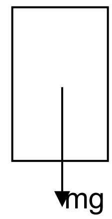
Some sample force diagrams of common situations are drawn below.

Mass on a string



Velocity v = 0, so T = mg Velocity v = constant upwards, so T = mg Velocity v = constant downwards, so T = mg Accelerating Upwards, T - mg = ma. Acceleration Downwards, mg - T = ma.

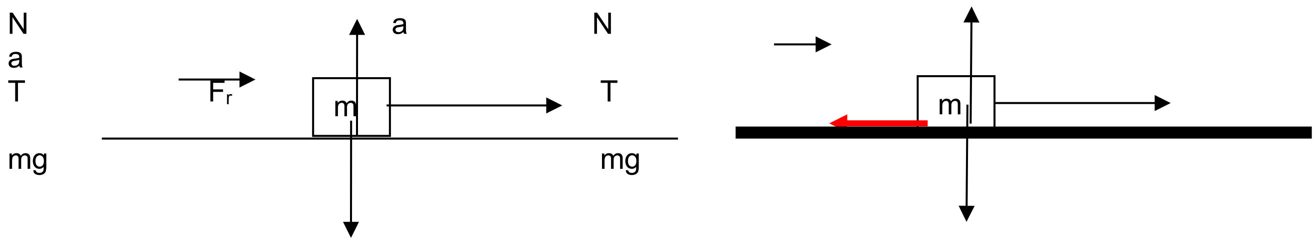
Mass in free flight



ΣF = mg = ma

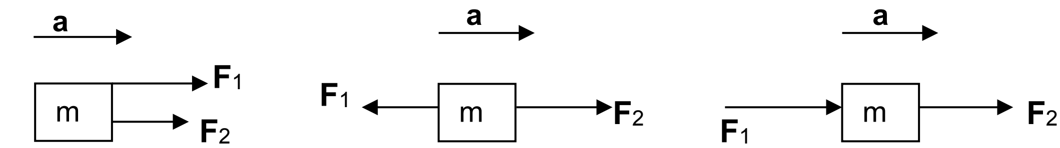
Mass pulled along a plane

Smooth (No Friction) Rough (Friction)



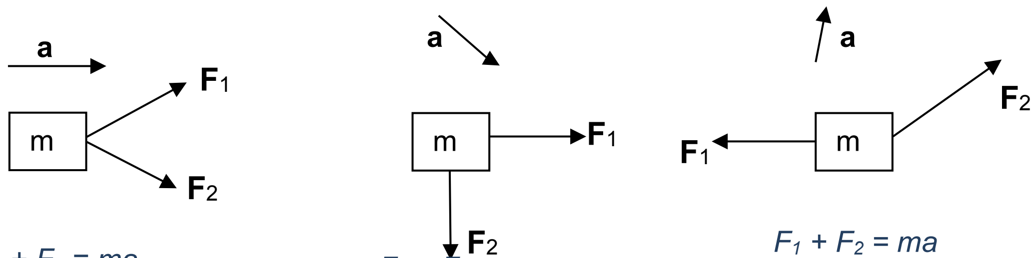
T = ma, **N** + mg = 0 T – $\mathbf{F}_{\mathbf{net}}$ = ma, **N** + mg = 0

Bodies with parallel forces acting



$F_1 + F_2 = ma$ *$F_2 – F_1 = ma$* *$F_1 + F_2 = ma$*

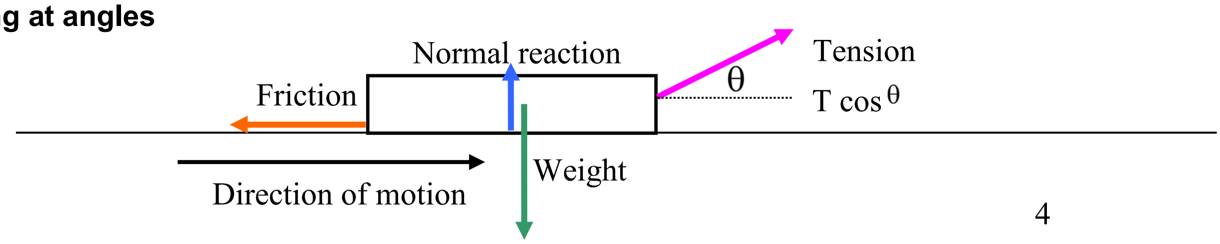
Bodies with non-parallel forces acting



$F_1 + F_2 = ma$ *$F_1 + F_2 = ma$* *$F_1 + F_2 = ma$*

The vectors need to be resolved in order to solve for the acceleration.

Forces acting at angles



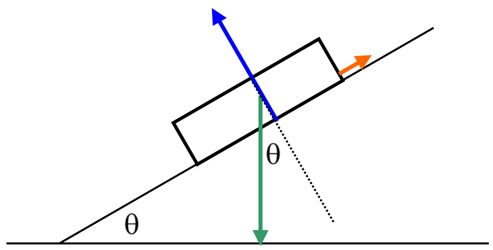
If this object is travelling at constant speed then the frictional force is equal to the horizontal component of the tension force.

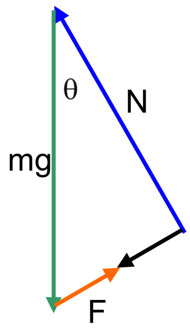
Inclined planes

Another example of forces acting at angles to each other is an object on an incline plane. There are only three different types of examples of a body on an incline plane without a driving force.

A body accelerating

The component of the weight force acting down the plane is larger then the frictional forces. (This is also true if there are no frictional forces). For these situations you would take down the plane to be positive, the reason for this is that the acceleration is down the plane.





Forces perpendicular to the plane

$F_{net} = mg\cos\theta - N$
$= 0$

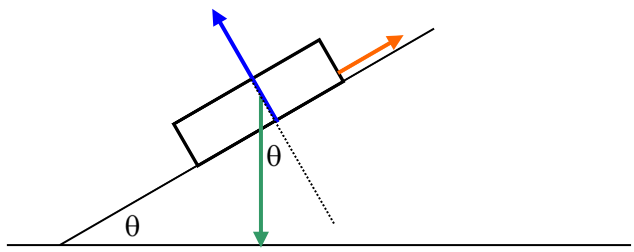
Forces parallel to the plane

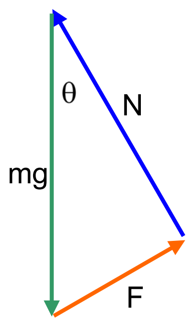
$F_{net} = mg\sin\theta - F$
$= ma$

Thus the acceleration is down the plane. If there is not friction then the acceleration is $g\sin\theta$

A body travelling at constant speed

This can be the when an object is not changing its speed whilst travelling down an incline or when the object is at rest on the incline plane.





Forces perpendicular to the plane

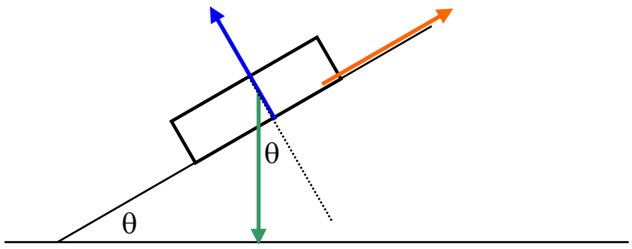
$F_{net} = mg\cos\theta - N$
$= 0$

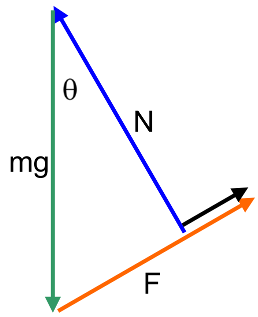
Forces parallel to the plane

$F_{net} = mg\sin\theta - F$
$= 0$

A body decelerating

For these situations you would choose up the plane to be positive, this is because this is the direction of acceleration.





Forces perpendicular to the plane

$$\mathbf{F_{net} = mg\cos\theta - N = 0}$$

Forces parallel to the plane

$$\mathbf{F_{net} = F - mg\sin\theta = ma}$$

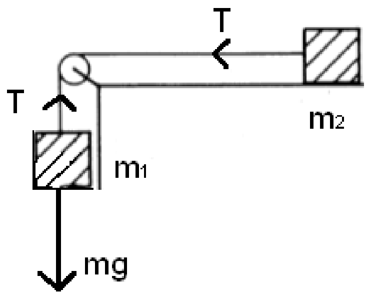
Thus the acceleration is up the plane.

Connected bodies

Problems involving the motion of two bodies connected by strings are solved on the following assumptions;

- the string is assumed light and inextensible so its weight can be neglected and
- there is no change in length as the tension varies.

In these cases, tension is the condition of a body subjected to equal but opposite forces which attempt to increase its length, and tension forces are pulls exerted by a string on the bodies to which it is attached.



To solve these problems you need to consider the vertical direction first.

$m_1g - T = m_1a$

The direction of this acceleration must be downwards.

This leads to: $T = m_1g - m_1a$

The tension in the string is the same in both directions, therefore $T = m_2a$.

Since both bodies are connected by an inextensible string, both bodies must have the same acceleration.

The vertical forces acting on m_2, (not shown) cancel each other out, and do not impact on its motion.

Combing these two equations gives $a = \frac{m_1}{m_1 + m_2}g$ $T = \frac{m_1m_2}{m_1 + m_2}g$

Hooke's Law

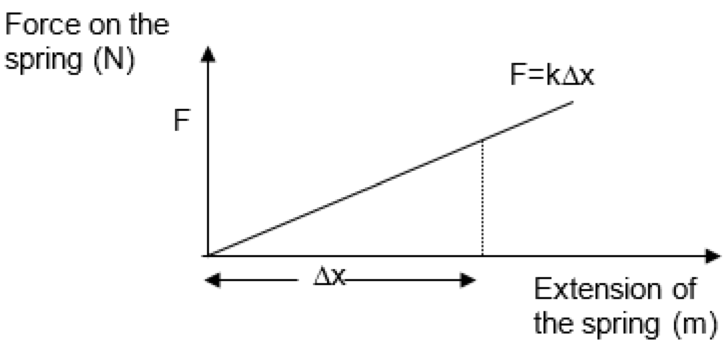
Extending a spring is an example of a situation where the force on an object is not constant. As the spring gets compressed, the force required to further compress it increases. The mathematical equation that represents this type of situation is called Hooke's Law

$$\boldsymbol{F = -k\Delta x}$$

Where F is the magnitude of the force required, Δx is the extension (or compression) of the spring, a k is called the spring constant. The minus sign in the equation is to show the vector relationship, the restoring force is in the opposite direction to the change in length. Typically we will not use the negative sign as the questions will just want the magnitude of the force.

The spring constant has a specific value for each individual spring. It depends on the size, thickness and material from which the spring is made. It is very important to use Δx, as this demonstrates that you are only interested in the **change in the length** of the spring.

The equation is illustrated in the graph below



The spring constant ‘k’ is the gradient of the line in the linear region. (Hooke’s law only holds in the linear region, before the material deforms as it goes beyond its elastic limit).

The spring constant has the units: Newtons per metre, ($N\ m^{-1}$).