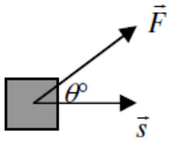
Work done by a constant force

Work done W by a constant force $\vec{F}$ on an object over a displacement $\vec{s}$ is defined as $W = Fs\cos\theta$.



If resolution of the force is in the same direction as displacement work in this situation is a positive amount and the object gains energy. If resolution of the force is in the opposite directions work is a negative amount and the object loses energy. Work is the change of energy. If object have energy it means it can do work. Work and energy are scalar quantities and they are measured in joules (J).

Example 1 A crate is pulled along a rough level surface with a rope at an angle of 25° above the horizontal. The tension in the rope is 100 N and friction force against motion is 80 N.
(a) Find the work done by the pulling force when the crate moves a distance of 1.2 m.
(b) Find the work done by the friction force over the 1.2 m.
(c) Find the work done by the normal force on the crate.
(d) Find the work done by the net force over the 1.2 m.

(a) $W = Fs\cos 25° = 100\times1.2\times\cos 25° = 110$ J (108.8 J).
(b) $W = -Fs = -80\times1.2 = -96$ J. The negative sign indicates that energy is taken out of the system.
(c) $W = Fs\cos 25° = N\times1.2\times\cos 90° = 0$, where N is the normal force on the crate.
(d) $F_{net} = 100 \times \cos 25 - 80 = 10.63\ N \quad W = 10.63 \times 1.2 = 12.756\,J = 13\,J$
Alternatively, $W = 108.8 - 96 = 12.8$ J = 13 J.

Example 2 How much work is done by the force of gravity of the earth on the moon in a month?

Force of gravity does no work on the moon because the force of gravity on the moon and the motion (velocity) of the moon are always perpendicular in direction.
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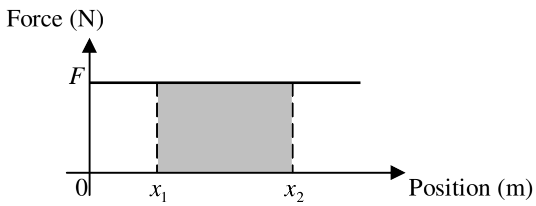
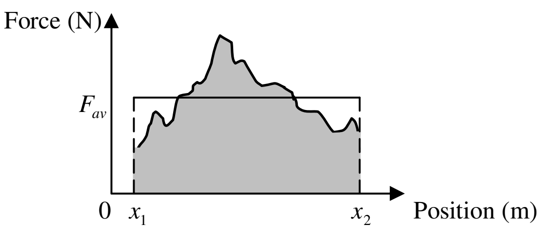
Area under force-position graph

When force $\vec{F}$ on an object is constant and in the same direction as $\vec{s}$, the rectangular area under the force-position graph from x_1 to x_2 represents the work done on the object by $\vec{F}$.



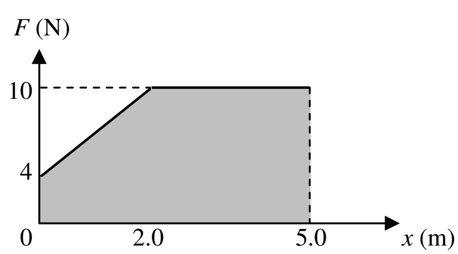
$W = area = F(x_2 - x_1) = Fs$.

When force $\vec{F}$ on an object changes with its position, area under the force-position graph still represents work done by $\vec{F}$ if $\vec{F}$ and $\vec{s}$ are in the same direction. Estimate the area if it cannot be determined by simple calculation.



The estimated area $\approx F_{av}(x_2 - x_1) = F_{av}s$.

Example 1 A 8.0-kg block moves in a straight line on a horizontal frictionless surface under the action of a force that varies with position (see graph below). How much work is done by the force as the block moves from the origin to $x = 5.0$ m?



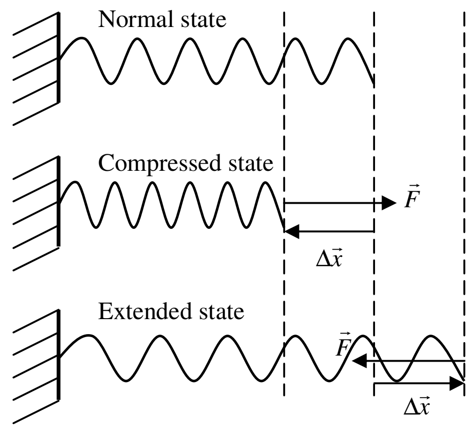
$W = area = 10 \times 5.0 - \frac{1}{2} \times 6 \times 2.0 = 44$ J.

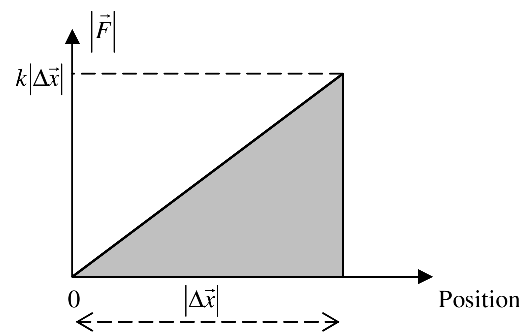
Hooke's law, an example of variable force

For an ideal spring the force exerted by the spring is directly proportional to its compression (or extension):

$$\vec{F} = -k\Delta\vec{x}$$

Note: In $\vec{F} = -k\Delta\vec{x}$, $\vec{F}$ is the force exerted by the spring, **not** the force in compressing or extending the spring. The minus sign indicates that $\vec{F}$ and $\Delta\vec{x}$ are in opposite directions. The proportionality constant k (Nm^{-1}) is called **spring constant**.





The area of the shaded region gives the work done in compressing or extending the spring by $|\Delta\vec{x}|$.

$$W = \frac{1}{2}k|\Delta\vec{x}|^2$$

Example 1 A spring is extended by 2.0 cm when a 5.0-kg load is suspended from it.
(a) Find the force required to stretch it by 5.0 cm.
(b) Determine the amount of work required to stretch it by 5.0 cm.
(c) Find the extra work required to stretch it by another 1.0 cm.

(a) $|\vec{F}| = k|\Delta\vec{x}|$, $5.0 \times 9.8 = k \times 0.020$, $k = 2450\,\text{Nm}^{-1}$.

$|\vec{F}| = 2450 \times 0.050 \approx 123$ N.

(b) $W = \frac{1}{2}k|\Delta\vec{x}|^2 = \frac{1}{2} \times 2450 \times 0.050^2 \approx 3.06$ J.

(c) $W = \frac{1}{2} \times 2450 \times 0.060^2 = 4.41$ J.

Extra work $\approx 4.41 - 3.06 = 1.35$ J.

Different kinds of energy

Elastic potential energy: Energy is stored in a spring when it is compressed or extended.
This energy is called **elastic potential energy**. For an ideal spring it is given by

$$E_{ep} = \frac{1}{2}k\left|\Delta\vec{x}\right|^2 .$$

It is equal to the work done in compressing or extending the spring by $\left|\Delta\vec{x}\right|$.

Kinetic energy: It is associated with the state of motion of an object. A moving object possesses kinetic energy E_k (J) and the amount depends on the mass m (kg) and the speed v (ms^{-1}) of the object. By definition,

$$E_k = \frac{1}{2}mv^2 .$$

The definition suggests that $E_k \propto m$ and $E_k \propto v^2$.

When the speed is the same, if the mass of an object is twice that of the other object, then it has twice the amount of kinetic energy than the other.

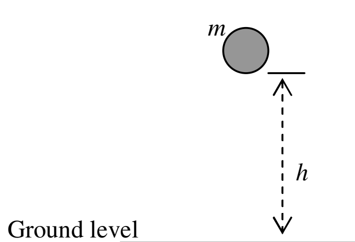
An object with twice the speed has four times the amount of kinetic energy.

Example 1 The speed of a car increases from 50 km h^{-1} to 100 km h^{-1}. Find the value of the ratio
kinetic energy at 100 km h^{-1} : kinetic energy at 50 km h^{-1}.

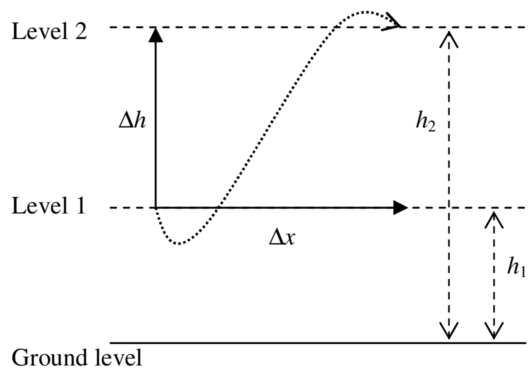
Since $E_k \propto v^2$, $\therefore \frac{E_{k,100}}{E_{k,50}} = \frac{100^2}{50^2} = 4$.

Gravitational potential energy: It is associated with the state of separation between objects that attract each other due to the gravitational force, e.g. between an apple and the earth.

The gravitational potential energy at ground level is arbitrary chosen as zero. By definition, the gravitational potential energy E_{gp} (J) of an object of mass m (kg) at a height h (m) above the ground is $E_{gp} = mgh$.



It is the vertical displacement that determines the change in gravitational potential energy. Horizontal displacement does not change the gravitational potential energy of an object.



When the object moves from level 1 to level 2, its gravitational potential energy increases, and

$$\Delta E_{gp} = mgh_2 - mgh_1 = mg(h_2 - h_1) = mg\Delta h .$$

When it moves from level 2 to level 1 its potential energy decreases by the same amount.

Example 1 A 70-kg person is carried by an escalator to the upper floor 4.5 m above.
(a) What is the increase in gravitational potential energy of the person?
(b) The person later takes a lift down to the lower floor. What is the decrease in her gravitational potential energy?

(a) $\Delta E_{gp} = mg\Delta h = 70\times 9.8\times 4.5 \approx 3.1\times 10^3$ J.

(b) Same amount.

The law of conservation of energy

Energy changes from one form to another and can be transferred from one object to another during interaction. Work is done in the transformation or transfer of energy, e.g. force of gravity does work on an object when the object falls, changing its gravitational potential energy to kinetic energy.

$$E_{gp} \xrightarrow{\text{Work}} E_k$$

The total amount of energy ($E_{gp} + E_k$) at any time during the fall is constant, i.e.

$$mgh_a + \frac{1}{2}mv_a{}^2 = mgh_b + \frac{1}{2}mv_b{}^2 \text{ or } \Delta E_k + \Delta E_{gp} = 0 .$$

This is known as the **law of conservation of energy**.

In the case of a moving object compressing a spring or extending a rubber cord, the law of conservation of energy can be expressed in terms of any two or all three of E_k, E_{ep} and E_{gp}.

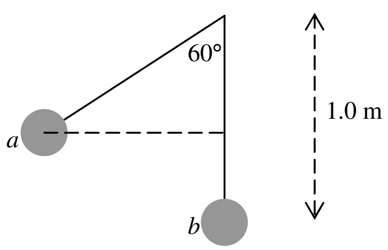
If only E_k and E_{ep} are involved in the situation,

$$\frac{1}{2}mv_a^2 + \frac{1}{2}k(\Delta x_a)^2 = \frac{1}{2}mv_b^2 + \frac{1}{2}k(\Delta x_b)^2 \quad \text{or}$$
$$\Delta E_{gp} + \Delta E_{ep} = 0.$$

If all three types of energy are involved,

$$mgh_a + \frac{1}{2}mv_a^2 + \frac{1}{2}k(\Delta x_a)^2 = mgh_b + \frac{1}{2}mv_b^2 + \frac{1}{2}k(\Delta x_b)^2 \quad \text{or}$$
$$\Delta E_k + \Delta E_{gp} + \Delta E_{ep} = 0.$$

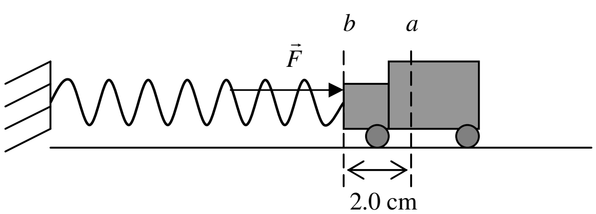
Example 1 A simple pendulum is released from position *a* to *b* (lowest point). Find the speed of the pendulum bob at *b*.



From *a* to *b* the distance fallen $= 1.0 - 1.0\cos 60° = 0.5$ m.

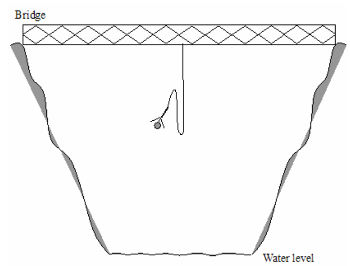
$$mgh_a + \frac{1}{2}mv_a^2 = mgh_b + \frac{1}{2}mv_b^2,$$
$$m(9.8)(0.5) = \frac{1}{2}mv_b^2, \quad v_b \approx 3.1\,\text{ms}^{-1}.$$

Example 2 A toy car (0.25 kg) moving at 1.5 ms^{-1} hits a spring, causing a maximum compression of 2.0 cm to the spring. Find the maximum force exerted by the spring on the toy car.



$$\frac{1}{2}mv_a^2 + \frac{1}{2}k(\Delta x_a)^2 = \frac{1}{2}mv_b^2 + \frac{1}{2}k(\Delta x_b)^2,$$
$$\frac{1}{2}(0.25)(1.5^2) = \frac{1}{2}k(^{-}0.020)^2, \quad \therefore k \approx 1.4\times10^3\ \text{Nm}^{-1}.$$
$$\vec{F} = -k\Delta\vec{x}, \quad \vec{F} = -(1.4\times10^3)(^{-}0.020) \approx {}^{+}28\ \text{N}.$$

Example 3 A 72.0-kg person attempts a bungee jump. The bungee cord is 35 m long and it is elastic (i.e. it follows Hooke's law) with $k = 250\,\text{Nm}^{-1}$. Air resistance is to be ignored. Consider the person as a point mass starting **from rest**, and just reaching the water.



(a) How high is the bridge above the water?
(b) Determine the speed of the person at 45 m below the bridge.

(a) Let *x* be the vertical distance between the bridge and the water. The person has gravitational potential energy at the bridge, and elastic potential energy at the water level.

$mgh_a + \frac{1}{2}k(\Delta x_a)^2 = mgh_b + \frac{1}{2}k(\Delta x_b)^2$, where *a* stands for at the bridge, and *b* at the water level.

$$72.0(9.8)x = \frac{1}{2}\times250(x-35)^2,$$
$$5.6448x = (x-35)^2, \quad x \approx 52\ \text{m}.$$

(b) $mgh_a + \frac{1}{2}mv_a^2 + \frac{1}{2}k(\Delta x_a)^2 = mgh_b + \frac{1}{2}mv_b^2 + \frac{1}{2}k(\Delta x_b)^2$, where *a* stands for at the bridge, and *b* at 45 m below the bridge.

$$72.0(9.8)(52) = 72.0(9.8)(52-45) + \frac{1}{2}(72.0)v_b^2 + \frac{1}{2}(250)(45-35)^2,$$
$$v_b \approx 23\ \text{ms}^{-1}.$$

Power

Power is a scalar quantity that measures the rate at which work is done by a force, or energy is transferred or transformed.

Average power is defined as $P_{av} = \frac{W}{\Delta t}$, or $P_{av} = \frac{\Delta E}{\Delta t}$, where W (J) is the amount of work done, ΔE (J) amount of energy transferred and Δt (s) the time taken.

Power is measured in joules per second or watts (J s^{-1}, or W).

Example 1 A load of bricks (420 kg) is to be lifted by a winch to a height of 20 m in 1.0 min. What must be the minimum power of the winch motor?

$$\Delta E_{gp} = 420\times9.8\times20 = 82320\ \text{J}$$
$$P = \frac{\Delta E}{\Delta t} = \frac{82320}{60} \approx 1.4\times10^3\ \text{W}$$

The actual power must be greater than this because friction and other retarding forces work against the lift. The winch is not 100% efficient. A fraction of the total amount of work done by the motor is used to lift the bricks, and the rest changes to heat and sound.

Efficiency

$$Efficiency = \frac{useful\ amount\ of\ work\ done}{total\ amount\ of\ work\ done} \times 100\%$$

or

$$Efficiency = \frac{useful\ energy\ transferred}{total\ energy\ used} \times 100\%$$

or

$$Efficiency = \frac{useful\ power}{total\ power} \times 100\%$$

Example 1 A motor produces 9000 J of heat while performing 2700 J of useful work. What is the efficiency of the motor?

Total energy output of motor = 2700 + 9000 = 11700 J.

$$Efficiency = \frac{2700}{11700} \times 100\% = 23\%$$

Example 2 A 38-percent-efficient power plant puts out 700 MW of electrical power.
(a) What is the rate of energy consumption of the power plant?
(b) How much heat is released into the atmosphere in an hour?

(a) $Efficiency = \frac{useful\ power}{total\ power} \times 100\%\ \ 38\% = \frac{700}{P_{total}} \times 100\%$
$P_{total} = 1.84 \times 10^3\ MW$

(b) In an hour total energy used
$E_{total} = P_{total} \times 1 = 1.84 \times 10^3\ MWh = 6.62 \times 10^9\ J$
Amount of heat $6.62 \times 10^9 \times 0.62 = 4.1 \times 10^9\ J$