2002

The Mars Odyssey spacecraft was launched from Earth on 7 April 2001 and arrived at Mars on 23 October 2001. Figure 1 is a graph of the gravitational force acting on the 700 kg Mars Odyssey spacecraft plotted against height above Earth's surface.

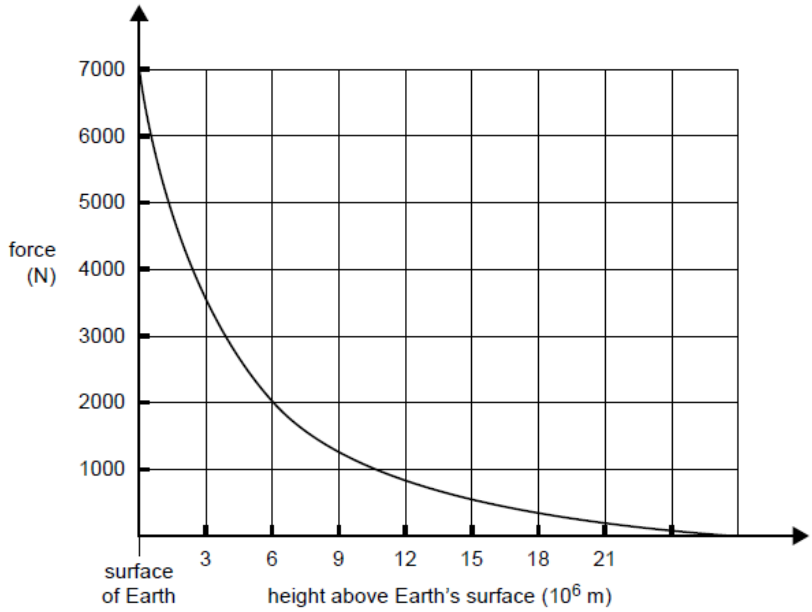


Figure 1

Question 1 (38%)

Estimate the minimum launch energy needed for Mars Odyssey to escape Earth's gravitational attraction. 3 marks

The required launch energy was calculated by determining the total area under the graph. Square counting resulted in approximately 13 squares, with each square representing a work done of 3 x 10^9 J. Hence, the total energy required was 13 x 3 x10^9 = 3.9 x 10^{10} J. Allowing for a variation in the number of squares counted, a range of values 3.3 to 4.4 x 10^{10} J, was accepted.

Most students recognised that the area under the graph was the key to answering this question. The most common error was incorrectly calculating the area of each square on the graph, usually by neglecting the 10^6 for the height axis. Others made an error in their estimation of the number of squares, usually in counting too few squares. Some students lost a mark due to multiplying their calculated area by 700 kg, obviously being confused between force and field.

2003

Figure 1 is a graph of the force of gravitational attraction between the 400 kg spacecraft Odyssey and the planet Mars versus distance above the surface of Mars.

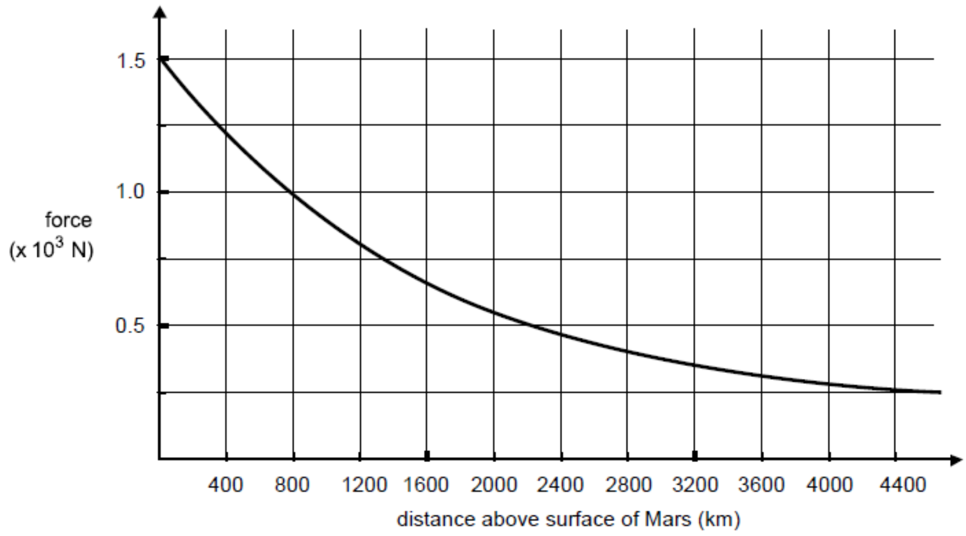


Figure 1

Before its orbit around Mars, Odyssey was originally launched from Earth and took 200 days to reach Mars. At a distance of 3200 km away from the surface of Mars it was travelling at approximately 24 000 m/s. At this point it was speeding up due to the gravitational attraction of Mars.

Question 2 (13%)

Describe, **but do not calculate**, the method you would use to determine the speed of Odyssey at a distance of 1200 km above the surface of Mars. 4 marks

Students needed to follow a logical sequence of steps in order to determine the speed of Odyssey. The steps were as follows:

• the gain in kinetic energy of Odyssey was equal to the area under the force-distance graph between 3200 km and 1200 km

• this results in the equation EKf – EKi = area under the graph

• this equation can be rewritten as $\frac{mv_f^2}{2} - \frac{mv_i^2}{2}$ = area under graph

• substitution of given values, along with the area calculation will result in a value for final velocity.

2004

Figure 1 shows the variation of gravitational field with height above Earth's surface.

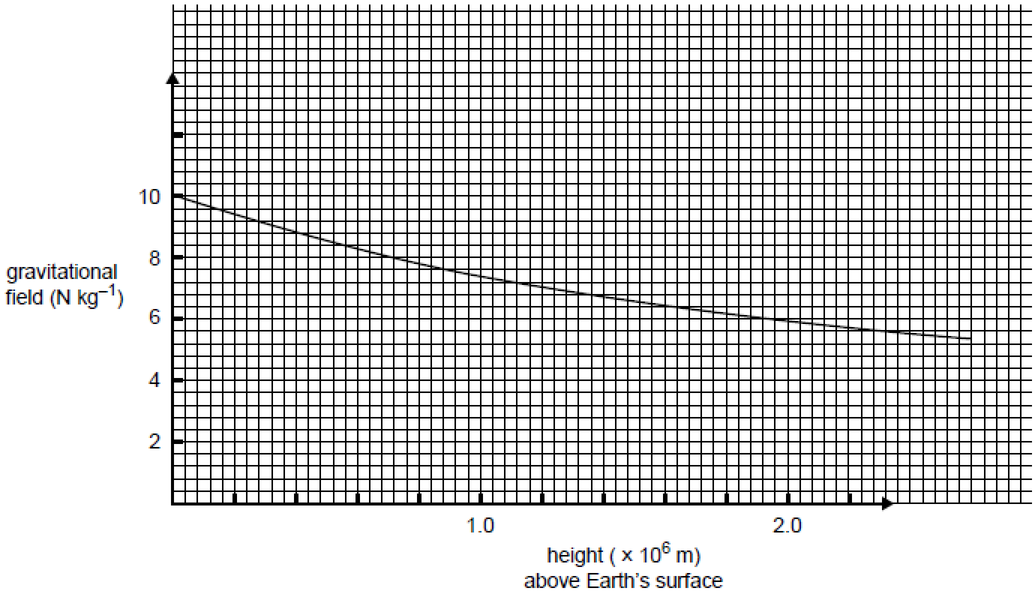


Figure 1

Question 3 (1%)

Calculate the energy needed to take the 400 kg spacecraft from rest at the surface of Earth and place it in a stable circular orbit of height 1.70×10^6 m. You must show your working.

5 marks

The energy needed to place the satellite in orbit was the sum of the gravitational potential energy needed to raise the satellite to the orbit height **and** the kinetic energy necessary for the orbit. The area under the graph represents the work done *per kilogram* to raise the satellite to this orbit height. Hence, the final expression for the energy required was:

Energy = 400 x area under graph + ½ $400v^2$.

v^2 could be found

$$F = \frac{mv^2}{r}$$

$$v^2 = \frac{Fr}{m} = \frac{mgr}{m} = gr$$

$v^2 = 6.3 \times (1.7 + 6.4) \times 10^6 = 5.1 \times 10^7$ (value of g=6.3 could be read from the graph)

This resulted in a total energy of approximately 1.55 x10^{10} J. A range of answers between 1.5 – 1.6 x10^{10} J was accepted.

✓**Question 9** (8 marks)

The spacecraft *Juno* has been put into orbit around Jupiter. The table below contains information about the planet Jupiter and the spacecraft *Juno*. Figure 11 shows gravitational field strength ($N\ kg^{-1}$) as a function of distance from the centre of Jupiter.

Data

mass of Jupiter	1.90×10^{27} kg
radius of Jupiter	7.00×10^{7} m
mass of spacecraft *Juno*	1500 kg

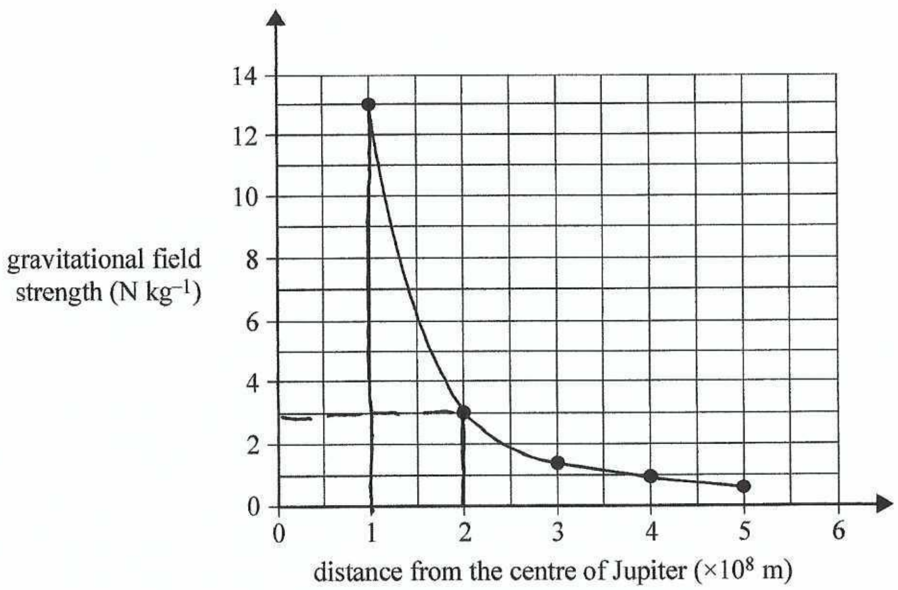


Figure 11

a. Calculate the gravitational force acting on *Juno* by Jupiter when *Juno* is at a distance of 2.0×10^{8} m from the centre of Jupiter. Show your working. 2 marks

57%

From the graph $g = 3\ N\ kg^{-1}$

$F_g = mg = 1500 \times 3 = 4500\ N$

4500 N

b. Use the graph in Figure 11 to estimate the magnitude of the change in gravitational potential energy of the spacecraft *Juno* as it moves from a distance of 2.0×10^8 m to a distance of 1.0×10^8 m from the centre of Jupiter. Show your working. 3 marks

33 %

ΔGPE = Area under the graph × mass

1 square $= 1 \times 0.5 \times 10^8 = 5 \times 10^7$ J kg^{-1}

14 squares $\Delta GPE = 14 \times 5 \times 10^7 \times 1500$

$= 1.05 \times 10^{12}$

J

$9 \times 10^{11} - 1.13 \times 10^{12}$ J

or

$\Delta GPE = mg_1h_1 - mg_2h_2$

$\approx 1500 \times 13 \times 10^8 - 1500 \times 3 \times 2 \times 10^8$

$= 1.05 \times 10^{12}$ J

✓**Question 4** (5 marks)

Assume that a journey from approximately 2 Earth radii ($2R_E$) down to the centre of Earth is possible. The radius of Earth (R_E) is 6.37×10^6 m. Assume that Earth is a sphere of constant density.

A graph of gravitational field strength versus distance from the centre of Earth is shown in Figure 4.

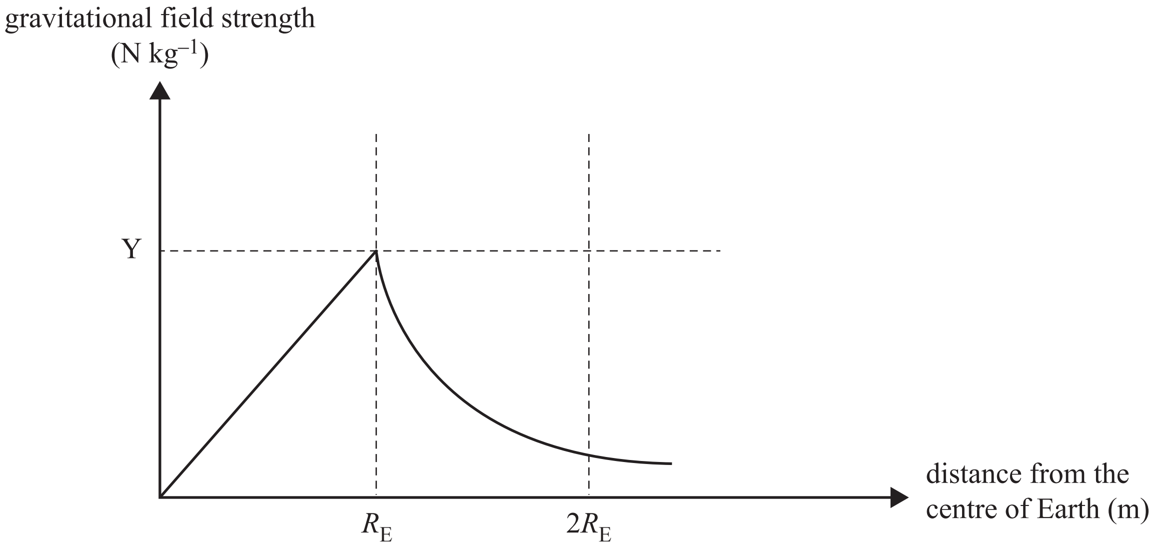


Figure 4

c. Calculate the increase in potential energy for a 75 kg person hypothetically moving from the centre of Earth to the surface of Earth. Show your working. 2 marks

25%

$E = m \times Area$

$= 75 \times \frac{1}{2} \times 6.37 \times 10^6 \times 9.8$

2.3×10^9 J

d. Figure 3 shows the strength of Earth's gravitational field, g, as a function of orbital altitude, h, above the surface of Earth.

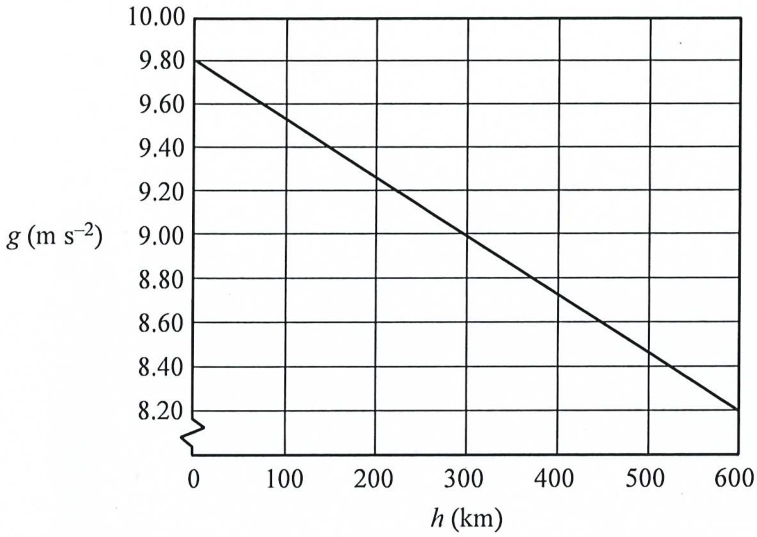


Figure 3

Determine the change in gravitational potential energy of the ICON satellite as it travels from Earth's surface to its orbital altitude of 600 km above Earth's surface. The mass of the ICON satellite is 288 kg.

3 marks

21%

ΔE_{gp} = Area under the graph 9.8 $\boxed{}$ 8.2 6×10^5 × mass

$= \frac{8.2+9.8}{2} \times 6\times10^5 \times 288 = 1.56\times10^9$

1.56×10^9 J

Question 8 (11 marks)

On 30 July 2020, the National Aeronautics and Space Administration (NASA) launched an Atlas rocket (Figure 7a) containing the Perseverance rover space capsule (Figure 7b) on a scientific mission to explore the geology and climate of Mars, and search for signs of ancient microbial life.

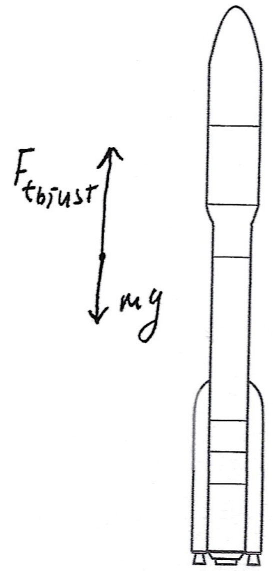


Figure 7a

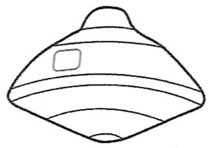


Figure 7b

a. At lift-off from launch, the acceleration of the rocket was 7.20 m s^{-2}. The total mass of the rocket and capsule at launch was 531 tonnes.

Calculate the magnitude and the direction of the thrust force on the rocket at launch. Take the gravitational field strength at the launch site to be $g = 9.80$ N kg^{-1}. Give your answer in meganewtons. Show your working.

3 marks

35%

$F_{thrust} - mg = ma$

$F_{thrust} = m(g+a)$

$= 5.31 \times 10^5 (9.8 + 7.2)$

$= 9.03 \times 10^6$

$= 9.03$ MN

9.03 MN	Up

On 18 February 2021, the Perseverance rover space capsule, travelling at 20 000 km h^{-1}, entered Mars's atmosphere at an altitude of 300 km above the surface of Mars. The mass of the capsule was 1000 kg.

b. Calculate the kinetic energy of the capsule at this point. Show your working. 2 marks

$v = 20000 \div 3.6 = 5556 \text{ ms}^{-1}$ 43%

$E_K = \frac{mv^2}{2}$

$E_K = \frac{1000 \times 5556^2}{2}$

$\boxed{1.54 \times 10^{10} \text{ J}}$

Figure 8 shows the gravitational field strength of Mars (g) versus altitude (h).

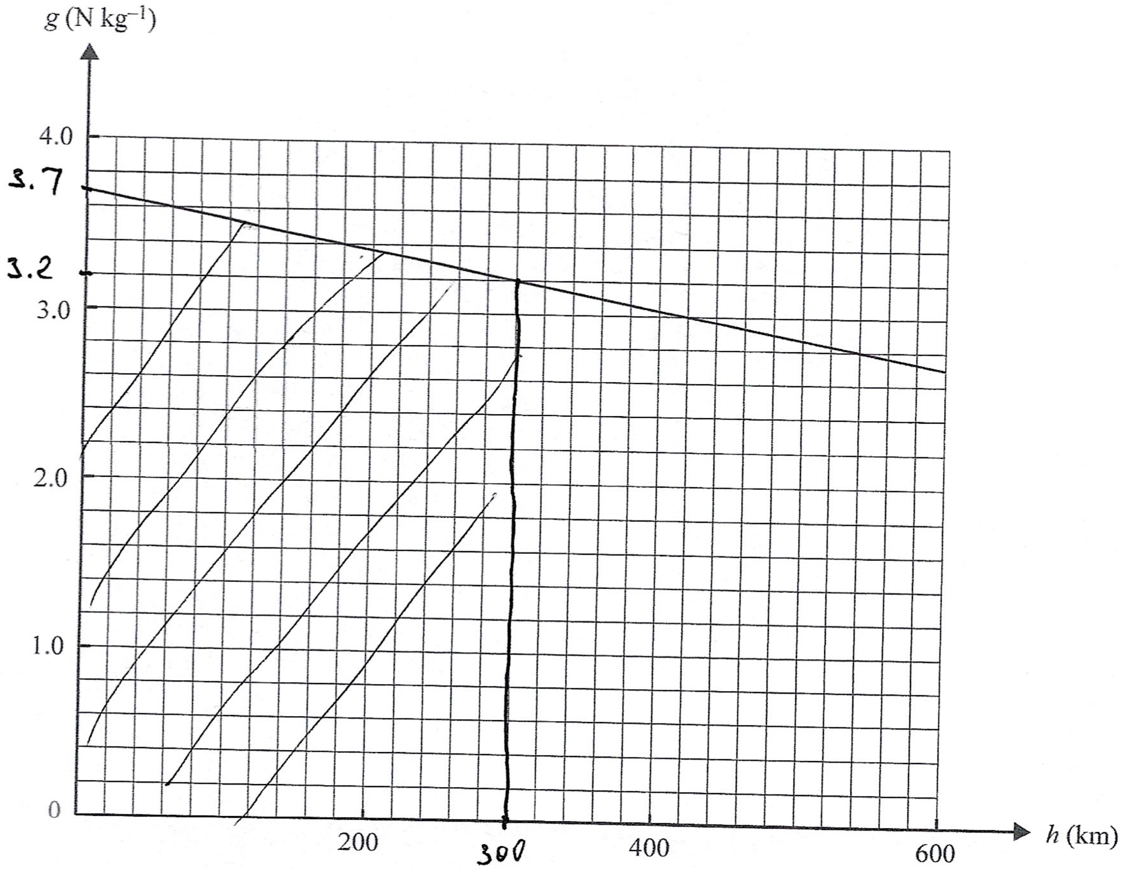


Figure 8

c. Calculate the gravitational potential energy of the capsule relative to the surface of Mars at an altitude of 300 km. Show your working. 3 marks

17%

E_{gp} = Shaded area on Figure 8 × mass
As squares are small and line is straight use formula for area of the trapezium

$E_g = 1000 \times \frac{3.7+3.2}{2} \times 300 \times 10^3$
$= 1.04 \times 10^9$ J

1 – 1.1 × 10⁹ was accepted

1.04×10^9 J

d. The capsule used aerodynamic braking as it descended through Mars's atmosphere to reduce its speed from 20 000 km h^{-1} to 1600 km h^{-1}. The capsule was then at an altitude of 10 km above the surface of Mars and had ~1% of its original combined gravitational potential energy and kinetic energy remaining.

Describe how ~99% of the gravitational potential energy and kinetic energy of the capsule was transformed and dissipated as the capsule descended from an altitude of 300 km above the surface of Mars to an altitude of 10 km above the surface of Mars. No calculations are required. 3 marks

6%

Energy is converted to heat (1m) due to air resistance (1m). When capsule descends its gravitational potential energy transforms into additional kinetic energy (1m)

Question 3

The gravitational field strength at the surface of a uniform spherical planet of radius R is g N kg^{-1}.
At a distance of $3R$ above the planet's surface, the strength of gravity will be closest to

A. 0

B. $\frac{g}{3}$

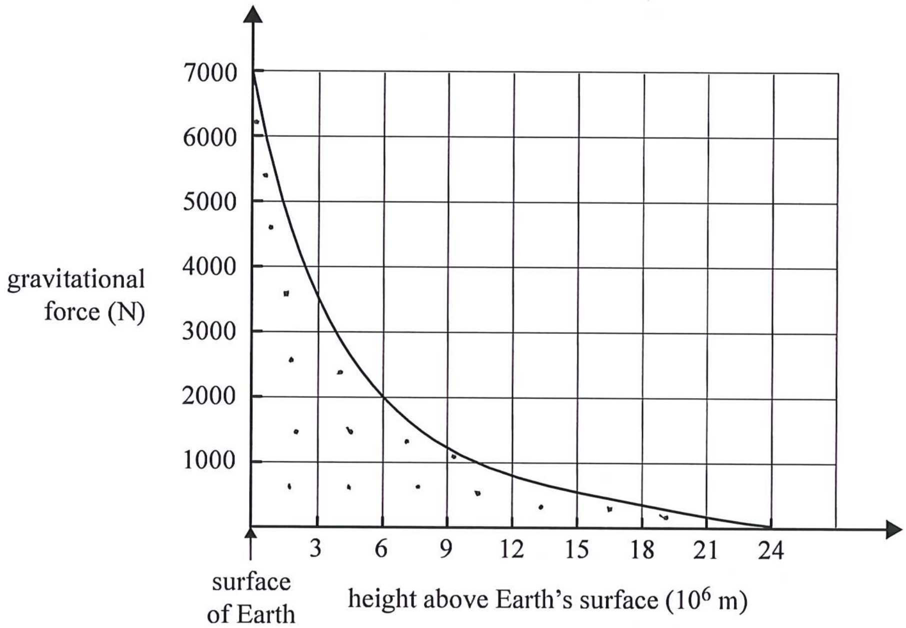
C. $\frac{g}{9}$

D. $\frac{g}{16}$

$g = \frac{GM}{R^2}$ $\quad g_1 = \frac{GM}{(4R)^2}$

Question 4

The Mars *Odyssey* spacecraft was launched from Earth to explore Mars. The graph below shows the gravitational force acting on the 700 kg Mars *Odyssey* spacecraft plotted against its height above Earth's surface.



Which one of the following is closest to the minimum launch energy needed for the Mars *Odyssey* spacecraft to 'escape' Earth's gravitational attraction?

A. 4.0×10^4 J

B. 1.5×10^5 J

C. 4.0×10^{10} J

D. 1.5×10^{11} J

1 sq $= 1000 \times 3 \times 10^6 = 3 \times 10^9$ J

11.5 sq $= 3.45 \times 10^{10}$

3.45×10^{10}

DO NOT WRITE IN THIS AREA

Question 6 (5 marks)

Measuring very small changes in Earth's surface mass, the 600 kg satellite GRACE-FO1 is in a circular orbit around Earth at an altitude of 500 km. The radius of Earth is 6.37×10^6 m.

a. Calculate the magnitude and direction of the satellite's centripetal acceleration. Give your answer correct to three significant figures. 3 marks

$g = \frac{GM}{r^2}$

$g = \frac{6.67\times10^{-11} \cancel{\times 6.37\times10^{6}}\ 5.98\times10^{24}}{(6.37 + 5\times10^5)^2}$ (1)

$= 8.45$

8.45 m s^{-2} (1) | Inwards (1)

b. Figure 6 shows a graph of the gravitational force that would act on GRACE-FO1 for a range of altitudes.

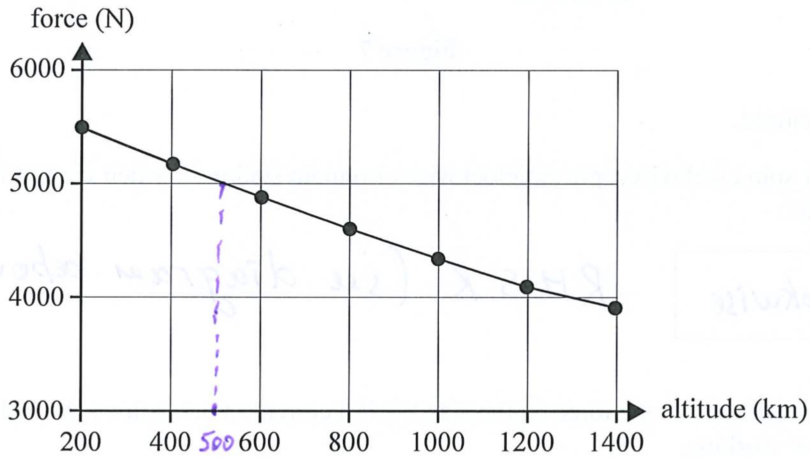


Figure 6

Estimate the energy required to lift the satellite from its present orbit at an altitude of 500 km to a new orbit at an altitude of 1400 km. 2 marks

$E_{gp} = \text{Area} = \frac{5000 + 3900}{2} \times 900 \times 10^3$

$= 4.0 \times 10^9$ (1)

$E_K = \frac{GMm}{2r}$ $\Delta E_K = \frac{GmM}{2}\left(\frac{1}{(6.87\times10^6)} - \frac{1}{(7.77\times10^6)}\right) =$

$= \frac{6.67\times10^{-11}\times600\times5.98\times10^{24}}{2}\left(\frac{1}{6.87\times10^6} - \frac{1}{7.77\times10^6}\right) =$

$= 2.02\times10^9$ J

$E = (4.0 - 2.02)\times10^9$

$= 1.98\times10^9$ J

1.98×10^9 J (1)

SECTION B – continued

TURN OVER