Distance contraction and time dilation

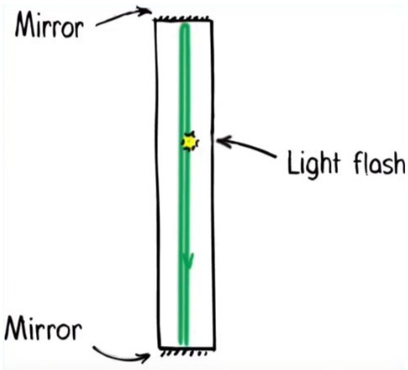
Lengths and durations measured in one frame of reference will produce difference values from the lengths and durations measured in a different frame of reference. Although all frames are equally valid, by convention the proper length and the proper time of an event are those measured in the frame of reference where the object or the clock is stationary.

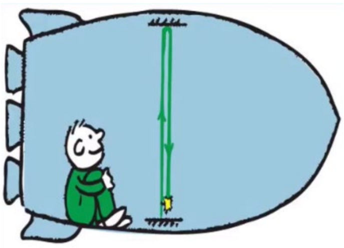
Proper Length: Length in the frame where the object (or distance) is stationary.
Proper Time: The time measured in the frame of reference where the clock is stationary.

Time Dilation

Imagine that we are able to somehow see a flash of light bouncing to and fro between two mirrors.

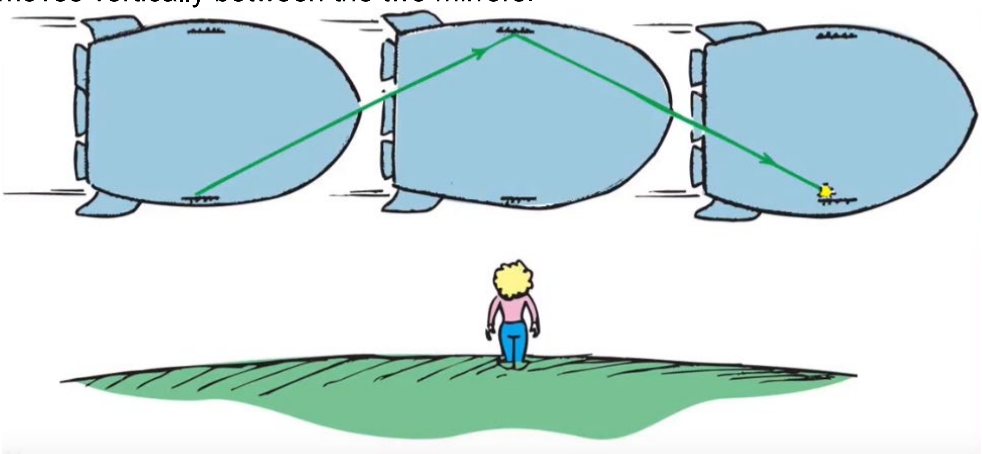
If the distance between the two mirrors is fixed then it will take the same time for the light to travel back and forth each time. We can call this a light clock.





Suppose that this light clock is inside a transparent high-speed spaceship. An observer who travels along with the ship, sees the light reflecting straight up and down between the two mirrors, just as it would if the spaceship was at rest.

Suppose now that we are standing on the ground as the space ship passes by us at high speed. From our reference frame we do not see the light path as being simply up and down motion, because it moves horizontally while it moves vertically between the two mirrors.

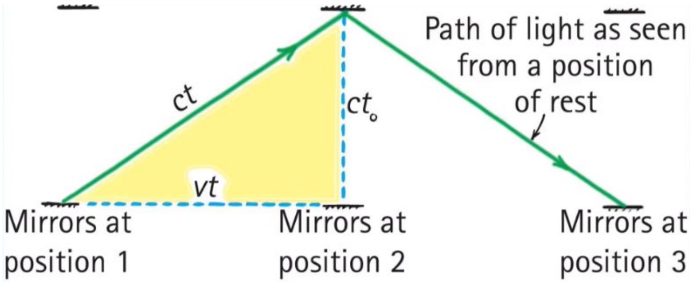


From the Earthbound frame of reference the flash travels a longer distanceas it makes its round trip between the two mirrors.

The speed is given by $\frac{\text{distance}}{\text{time}}$ and since the speed of light is the same in all refernce frames (Einstein's second posulate) then $c = \frac{\text{distance}}{\text{time}}$.

If the distance in the Earthbound frame of refernce is larger, then the time taken for the flash to make a round trip must also be longer.(The longer diagonal distance must be divided by a correspondinlgy longer time interval to maintain an unvarying value for the speed of light.)
This stretching of time is called **time dilation.**

If we consider the path of the light as it travels from position 1 to position 2 horizontally.
The person in the spaceship (rest frame) sees the light travel vertically from one mirror to the other. (mirrors at position 2). The distance that the light travels is given by $d = v \times t$
$\therefore ct_0$, where t_0 is the time it takes for the flash to move between mirrors as measured from a frame of reference fixed to the light clock (rest frame)



From the frame of reference (Earth) where the lightclock moves with a speed 'v', the horizontal distance the mirror moves is given by 'vt', where t is the time, as measured from the Earth. (The time meaured of a moving clock). The path the light travels is the diagonal from the the mirror at position 1 to the other mirror at position 2. This distance is given by 'ct'.

Using Pythagoras in the yellow triangle we get

$$(ct)^2 = (vt)^2 + (ct_0)^2$$
$$\therefore c^2t^2 = v^2t^2 + c^2t_0^2$$
$$\therefore c^2t^2 - v^2t^2 = c^2t_0^2$$
$$\therefore t^2(c^2 - v^2) = c^2t_0^2$$
$$\therefore t^2\left(\frac{c^2 - v^2}{c^2}\right) = t_0^2$$
$$\therefore t\sqrt{\left(\frac{c^2 - v^2}{c^2}\right)} = t_0$$
$$\therefore t\sqrt{1-\frac{v^2}{c^2}} = t_0$$
$$\therefore t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where:

t = elapsed time in moving frame

t_0 = time in a stationary frame

Typically we call the expression $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, gamma, γ.

$$\therefore \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

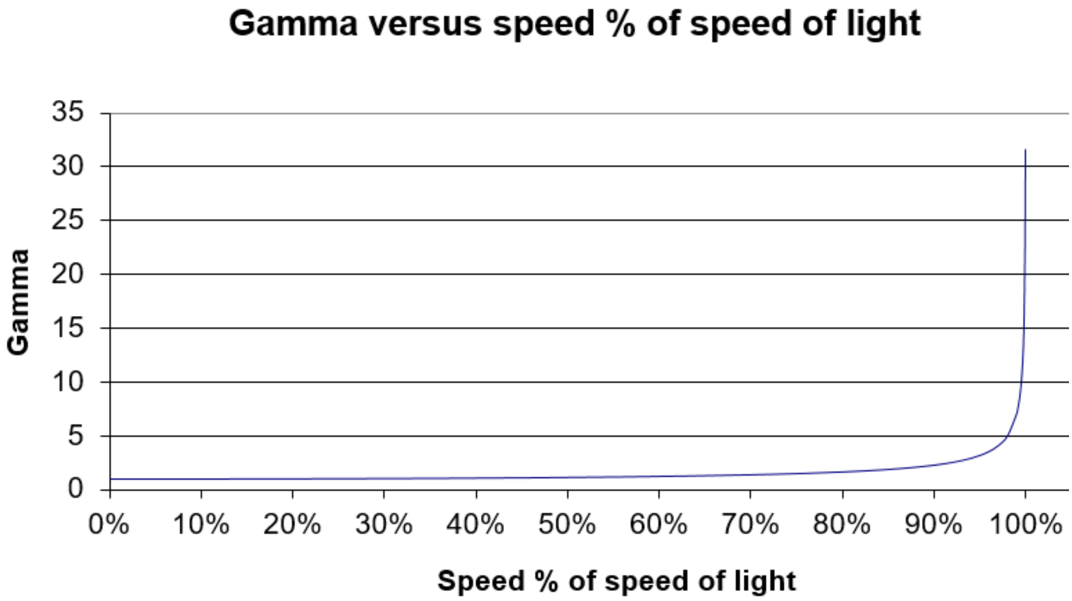
Therefore, the relative time is $\mathbf{t = \gamma t_0}$.   Note that t_0 is the proper time between the two events.

γ will always be greater than 1, so t is always greater than t_0. We think about this as **the time measured in the rest frame is always the smallest.**

Gamma

Lengths, times and masses are different in different frames of reference. The quantity ***gamma*** γ is used to determine the changes

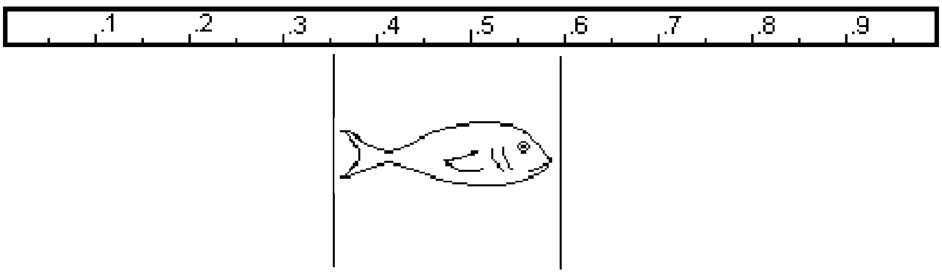
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Length Contraction

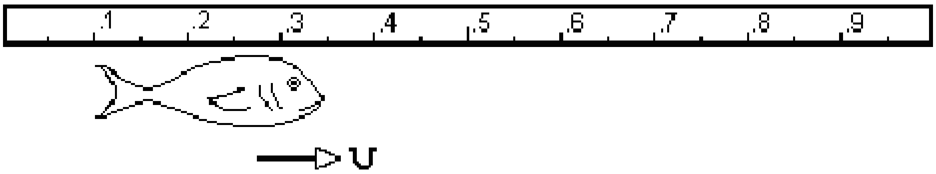
The length of a moving object will be measured to be less than the length of the same object measured in the frame when the object is stationary. This is ***length contraction***.

To measure the length of an object, we place it next to the ruler, and measure the two endpoints. In this case, we see that the fish is between the points 0.35 and 0.6. By finding the distance between the two endpoints, we find the length of the fish.   0.6 – 0.35 gives us 0.25 units.

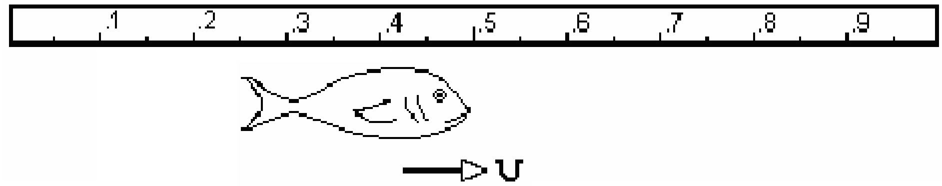


Things become more complex when the object is moving. The problem is that when the object is moving, the endpoints must be measured at exactly the same time.

Let's take an example. A space-fish is travelling along at great speed (>0.3c). We measure the point where the head is:



And find it is at 0.35 units. A moment later, we measure the point where the tail is:



The tail is at 0.25 units. Using the same calculation as before, we find the fish is 0.35 – 0.25 = 0.1 units. Now this is clearly wrong, because we are not measuring the two points simultaneously.

The problem with measuring length is that it requires two events (measuring each endpoint) to be simultaneous, but simultaneity will never be agreed upon by two observers in different frames of reference.

In order to measure the **proper** length of an object, it must be measured from a frame of reference in which it is at rest. If it is being measured as it is moving, we can use the following formula to calculate its **relativistic length contraction**.

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} \qquad \text{or} \quad L = \frac{L_0}{\gamma}$$

The 'proper' length of an object is the length measured in a reference frame in which the object is at rest.
In the rest frame, the length is longest, it contracts when in moving frames

Eg. What velocity must an observer have to observe a length contraction of 50%.

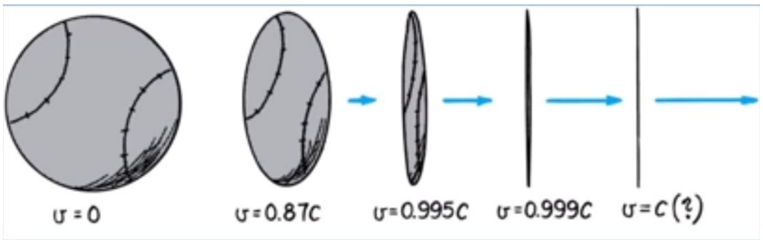
$$\frac{L}{L'} = \frac{1}{2} = \sqrt{\left[1 - \frac{v^2}{c^2}\right]}$$

$$\therefore \frac{1}{4} = 1 - \frac{v^2}{c^2}$$

Hence $$\therefore \frac{v^2}{c^2} = \frac{3}{4} \times \left(3 \times 10^8\right)^2$$

$$v = 0.866c$$
$$= 2.60 \times 10^8 \text{ m s}^{-1}$$

The length only contracts in the direction of motion, and as speed increases, length contraction increases.



The cosmic muon

Muons are elementary particles that can be created in the laboratory where they are nearly at rest. Their average lifetime under these circumstances (proper time) $\Delta t' = 2.2\ \mu s$. (Note that this time is measured in a frame of reference in which the observer is stationary with respect to the muon)

Muons are also created high (several kilometres) in the atmosphere by the action of cosmic rays and are then travelling with a typical velocity of v = 0.99 c. Using classical Newtonian mechanics we would calculate the average distance that a muon could travel as **2.2 μs x 0.99 c = 650 m**. This would suggest that only a few muons would ever reach the earth's surface. BUT muons from the upper atmosphere reach the earth's surface in abundance. To resolve this inconsistency use relativistic (rather than Newtonian) mechanics

Consider the moving muon to be a 2.2 μs 'clock' travelling at 0.99 c.
To an observer on earth the lifetime of the muon would seem to be

$$\Delta t = \gamma \Delta t'$$
$$= \frac{2.2 \times 10^{-6}}{\sqrt{\left[1 - \frac{0.99c^2}{c^2}\right]}}$$
$$= 16\ \mu s$$

In this period of time the muon can travel **0.99 c x 16 x 10^{-6} = 4600 m**.

e.g. The period of a pendulum is measured to be 3.0 s in the inertial frame of the pendulum. What is the period of the pendulum measured by an observer moving at 0.95 c with respect to the pendulum?

'Proper' time = $\Delta t'$ = 3 s.

$$t = \gamma t' = \frac{1}{\sqrt{\left[1 - \left(\frac{0.95c}{c}\right)^2\right]}} \times 3.0$$
$$= 3.2 \times 3.0$$
$$= 9.6\, s$$

i.e. the moving observer observes the pendulum to oscillate more slowly.

What distance, in our frame of reference, represents 650 m in the μ meson's frame of reference?

$$L = 650\text{m}$$

Hence $$L' = \frac{650}{\sqrt{1 - \left(\frac{0.99c}{c}\right)^2}}$$

$$= \underline{4{,}600\,\text{m}} \quad \text{as before}$$