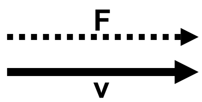
Circular Motion notes and

how to solve problems with circular motion

According to the 2nd Newtons law whenever there is a non-zero resultant force, then there will be acceleration in the direction of the net force, which will produce a change in velocity.

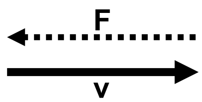
Consider the following 5 cases.

Case 1



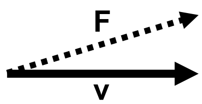
the object will speed up, but there will not be a change of direction

Case 2



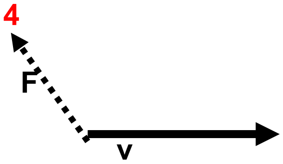
the object will slow down, but there will not be a change of direction

Case 3



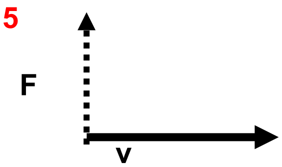
the object will speed up, with a change of direction

Case 4



the object slows down, with a change of direction

Case 5

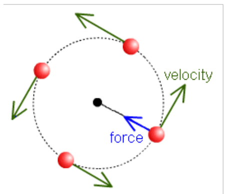


No change of speed, but a change of direction

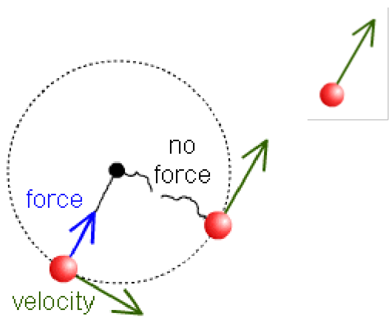
In all the cases above, there is acceleration, because there is a net force acting. But this acceleration does not always result in a change of speed.
Remember that velocity is a vector, and has a magnitude ***and*** a direction.

Object will be moving in a circle if the **net** force is at the right angle to the velocity (in this case object moving with constant speed as force changing only the direction of the velocity) or at least have component perpendicular to the velocity (in this case magnitude of the velocity changing as well). Centripetal force is a **net** force. There is no such separate force in the nature.

When a body travels in a circle, the force (and acceleration) is towards the centre while the velocity is tangential. The force is always at right angles to the velocity. The force is sometimes provided by a string or rope, but it could be supplied by friction (cars going around corners, or people running on circular paths) or gravity (the moon and other satellites around the earth).



If the cord breaks the force is removed and the circular motion ceases. The object will fly off in the direction it was travelling at the time of the break.



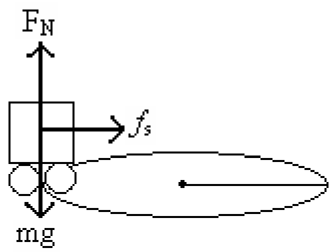
The formula for the magnitude of a net force necessary to keep an object in circular motion (such force is called centripetal force. Centripetal force is a **net** force. There is no such separate force in the nature.) is: $F_c = ma_c = m\frac{v^2}{r}$,

where **v** is the magnitude of the velocity (speed) of the object, **r** is the radius of the circle and $\boldsymbol{a_c}$ is the centripetal acceleration always directed to the centre of the circle at the right angle to the velocity.

Solving problems with circular motion require same approach as solving problems with the forces, so the same steps must be followed.

Most of the problems involving circular motion could be divided into 4 groups: horisontal circle on the ground, horisontal circle of pendulum, vertical circle, circle on banked road.

Horisontal circle on the ground.



$$\Sigma F_c = ma_c \qquad \Sigma F_y = ma_y$$

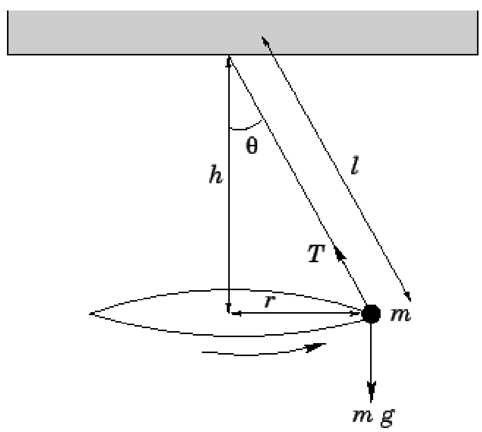
$$f_s = ma_c \qquad F_N = mg$$

$$f_s = \frac{mV^2}{R}$$

$$f_{smax} = \mu_s F_N = \frac{mV^2}{R}$$

In this case friction force is the net force, so centripetal force equal friction force and perpendicular to the velocity. If $F_{fr\ max} < \frac{mv^2}{r}$ object will slide away from the centre until increase in radius will make them equal again (remember this when you will be driving the car!).

Horisontal circle of the pendulum.



In this case net force (centripetal force) $F_c = T\sin\theta$ (resolution of the second Newton's law in the horisontal direction) and $T\cos\theta - mg = 0$ (resolution of the second Newton's law in the vertical direction). $F_c = \frac{mv^2}{r}$

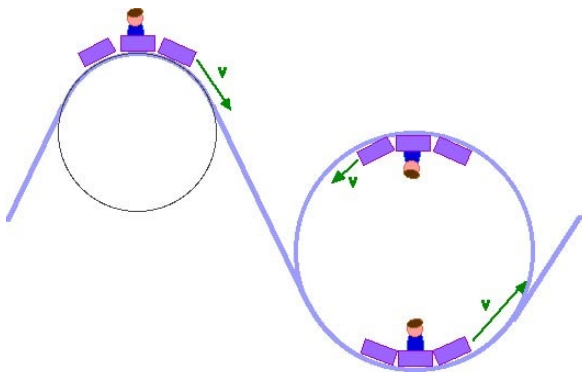
$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

Solving these 2 simultaneous equations will give us unknown quantity.

Vertical circle

Car on/under the bridge, in the dip.



To solve those problems we should write resolution of the second Newton's law into the vertical direction.

In the first case (car at the top of the bridge) $mg - N = \frac{mv^2}{r}$ so $N = mg - \frac{mv^2}{r}$

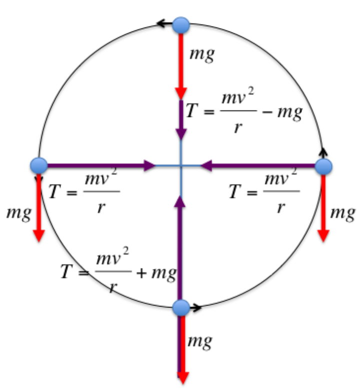
The **maximum** speed that car can have without losing traction with the road will be when normal reaction becomes equal 0, which gives $v = \sqrt{gr}$

In the second case when car is at the top of the circle but under the track $N + mg = \frac{mv^2}{r}$

The **minimum** speed car can have without falling will be when normal reaction becomes equal 0, which gives $v = \sqrt{gr}$

In the third case (car is at the bottom of the circle) $N - mg = \frac{mv^2}{r}$ so $N = mg + \frac{mv^2}{r}$

Mass on the string.

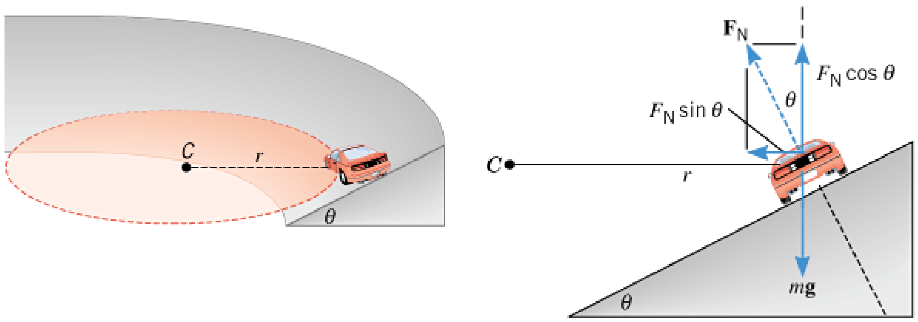


Tension at the bottom is greater than at the top.

Top: $T + mg = \frac{mv^2}{r}$ $\qquad T = \frac{mv^2}{r} - mg$

Bottom: : $T - mg = \frac{mv^2}{r}$ $\quad T = \frac{mv^2}{r} + mg$

Banked road



(a) (b)

In this case we should choose one axis vertical and one horisontal. Consider situation when frictional force equal 0.

Resolution of the second Newton's law into vertical direction $N\cos\theta - mg = 0$,

Into the horisontal $N\sin\theta = \frac{mv^2}{r}$

Note that normal reaction in this case is greater compare to the case when object sliding down the inclined plane.

Solving these equation gives $v = \sqrt{gr\tan\theta}$

This is the speed required to move without friction involved. If speed will exceed this value object will be sliding up to increase radius and equalize sides of equation again, so friction will be directed down the plane. If speed will be smaller than vise versa.

Other Formulae

The period (T) of an orbit is the time it takes for an object to complete one full circuit. We can use the fact that the speed of the object v is constant and the distance the object travels during one orbit is the circumference of the circle, $2\pi r$ to derive the formula

$$T = \frac{2\pi r}{v}$$

Rearranging $v = \frac{2\pi r}{T}$ and substituting in the centripetal force formula we will get $F_c = \frac{4\pi^2 rm}{T^2}$